Problem statement A continuous function f is defined on the interval $[-2, 2]$. The values of f at some of the points of the interval are given by the following table:

x	-2	-1	0	1	2
$f(x)$	2	-1	2	-1	2

a) Using only this information, what can be concluded about the roots of f, that is, the solutions of $f(x) = 0$, in the interval $[-2, 2]$? The answer should be something like: f has at least 8 roots in $[-2, 2]$, or f has at most 6 roots in $[-2, 2]$.

Suggestion Use the Intermediate Value Theorem on each of the intervals $[-2, -1]$, $[-1, 0]$, $[0, 1]$, and $[1, 2]$.

b) If $f(x) = x^4 - 4x^2 + 2$, verify that the relevant values of f are given by the table above.

i) Sketch the graph of $y = f(x)$ in the viewing window $[-2.5, 2.5]\times[-3, 3]$.

ii) How many roots does f have in the interval $[-2, 2]$? Find the roots algebraically. *Suggestion:* Let $t = x^2$ and solve with the quadratic formula. Then find x.

c) If $f(x) = x^4 - 4x^2 + 2 + 5(2x - 1)x(x^2 - 1)(x^2 - 4)$. Verify that the relevant values of f are given by the table above.

i) Sketch the graph of $y = f(x)$ in the viewing window $[-2.5, 2.5]\times[-80, 80]$.

ii) Explain why f has at least one root in each of the intervals $(-2, 1)$, $(-1, 0)$, $(0, 1)$, and $(1, 2)$.

iii) Sketch the graph of $y = f(x)$ in the viewing window $[0, 1]\times[-1, 3]$.

iv) How many roots does f have in the interval $[0, 1]$?

Approximate the roots of f in $[0, 1]$ to three decimal places using a calculator.

d) Having done b) and c), was your original conclusion in part a) correct?

Problem statement a) Suppose $f(x) = 3^x$. Plot $y = f(x)$ in the square window defined by $-1 \le x \le 1$ and $0 \le y \le 2$. Also plot the secant lines connecting $(0, f(0))$ and $(0 + h, f(0 + h))$ for $h = .5$ and $h = .25$ in the same window. Give a table of values of the slope of the secant lines connecting $(0, f(0))$ and $\left(10^{-j}, f\left(10^{-j}\right)\right)$ when j is a positive integer ranging from 1 to 5. What is an equation of the line tangent to $y = 3^x$ at $(0, 1)$?

b) Suppose $g(x) = 6x \arctan\left(\dfrac{\ln x}{x^3 + 2}\right)$. Plot $y = g(x)$ in the square window defined by $0 \le x \le 2$ and $-1 \le y \le 1$. Also plot the secant lines connecting $(1, g(1))$ and $(1 + h, g(1 + h))$ for $h = .5$ and $h = .25$ in the same window. Give a table of values of the slope of the secant lines connecting $(1, g(1))$ and $\left(1 + 10^{-j}, g\left(1 + 10^{-j}\right)\right)$ when j is a positive integer ranging from 1 to 5. What is an equation of the line tangent to $y = 6x \arctan\left(\dfrac{\ln x}{x^3 + 2}\right)$ at $(1, 0)$?

Problem statement Suppose that A, B, C, and D are constants and f is the cubic polynomial $f(x) = Ax^3 + Bx^2 + Cx + D$. Suppose also that the tangent line to $y = f(x)$ at $x = 0$ is $y = x$ and the tangent line at $x = 2$ is given by $y = 2x - 3$. Find the values of A, B, C, and D. Then sketch the graph of $y = f(x)$ and the two tangent lines for $-2 \le x \le 4$.

Problem statement Some lines which are tangent to the parabola $y = x^2$ also pass through the point $(2, 3)$. Find all of these lines. Graph the parabola and the tangent lines which were found on the same coordinate axes.