**Problem statement** A continuous function \( f \) is defined on the interval \([-2, 2]\). The values of \( f \) at some of the points of the interval are given by the following table:

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>-1</td>
<td>2</td>
<td>-1</td>
<td>2</td>
</tr>
</tbody>
</table>

a) Using only this information, what can be concluded about the roots of \( f \), that is, the solutions of \( f(x) = 0 \), in the interval \([-2, 2]\)? The answer should be something like: \( f \) has at least 8 roots in \([-2, 2]\), or \( f \) has at most 6 roots in \([-2, 2]\).

**Suggestion** Use the Intermediate Value Theorem on each of the intervals \([-2, -1]\), \([-1, 0]\), \([0, 1]\), and \([1, 2]\).

b) If \( f(x) = x^4 - 4x^2 + 2 \), verify that the relevant values of \( f \) are given by the table above.

   i) Sketch the graph of \( y = f(x) \) in the viewing window \([-2.5, 2.5] \times [-3, 3]\).
   
   ii) How many roots does \( f \) have in the interval \([-2, 2]\)? Find the roots algebraically.

   **Suggestion:** Let \( t = x^2 \) and solve with the quadratic formula. Then find \( x \).

c) If \( f(x) = x^4 - 4x^2 + 2 + 5(2x - 1)x(x^2 - 1)(x^2 - 4) \). Verify that the relevant values of \( f \) are given by the table above.

   i) Sketch the graph of \( y = f(x) \) in the viewing window \([-2.5, 2.5] \times [-80, 80]\).
   
   ii) Explain why \( f \) has at least one root in each of the intervals \((-2, 1), (-1, 0), (0, 1), \) and \((1, 2)\).

   iii) Sketch the graph of \( y = f(x) \) in the viewing window \([0, 1] \times [-1, 3]\).
   
   iv) How many roots does \( f \) have in the interval \([0, 1]\)?

Approximate the roots of \( f \) in \([0, 1]\) to three decimal places using a calculator.

d) Having done b) and c), was your original conclusion in part a) correct?

**Problem statement** a) Suppose \( f(x) = 3^x \). Plot \( y = f(x) \) in the square window defined by \(-1 \leq x \leq 1\) and \(0 \leq y \leq 2\). Also plot the secant lines connecting \((0, f(0))\) and \((0 + h, f(0 + h))\) for \(h = .5\) and \(h = .25\) in the same window. Give a table of values of the slope of the secant lines connecting \((0, f(0))\) and \((10^{-j}, f(10^{-j}))\) when \(j\) is a positive integer ranging from 1 to 5. What is an equation of the line tangent to \( y = 3^x \) at \((0, 1)\)?

b) Suppose \( g(x) = 6x \arctan\left(\frac{\ln x}{x^3 + 2}\right) \). Plot \( y = g(x) \) in the square window defined by \(0 \leq x \leq 2\) and \(-1 \leq y \leq 1\). Also plot the secant lines connecting \((1, g(1))\) and \((1 + h, g(1 + h))\) for \(h = .5\) and \(h = .25\) in the same window. Give a table of values of the slope of the secant lines connecting \((1, g(1))\) and \((10^{-j}, g(10^{-j}))\) when \(j\) is a positive integer ranging from 1 to 5. What is an equation of the line tangent to \( y = 6x \arctan\left(\frac{\ln x}{x^3 + 2}\right) \) at \((1, 0)\)?

**Problem statement** Suppose that \( A, B, C, \) and \( D \) are constants and \( f \) is the cubic polynomial \( f(x) = Ax^3 + Bx^2 + Cx + D \). Suppose also that the tangent line to \( y = f(x) \) at \( x = 0 \) is \( y = x \) and the tangent line at \( x = 2 \) is given by \( y = 2x - 3 \). Find the values of \( A, B, C, \) and \( D \). Then sketch the graph of \( y = f(x) \) and the two tangent lines for \(-2 \leq x \leq 4\).

**Problem statement** Some lines which are tangent to the parabola \( y = x^2 \) also pass through the point \((2, 3)\). Find all of these lines. Graph the parabola and the tangent lines which were found on the same coordinate axes.