REVIEW PROBLEMS FOR THE FINAL EXAM
MATH 151, 2015

1. For what values of B is the following function continuous at $x = 1$?

$$f(x) = \begin{cases} 3x^3 - x^2 - Bx, & x > 1 \\ Bx - 2, & x \leq 1. \end{cases}$$

2. Find the equation of the tangent line to the curve $y = \dfrac{x-1}{x+1}$ at each point where the tangent is parallel to the line $x - 2y = 2$.

3. Find the equation of the tangent line to the curve defined by the equation

$$\ln(xy) + 2x - y + 1 = 0$$

at the point $(\frac{1}{2}, 2)$.

4. Find the following limits:

a) $\displaystyle\lim_{x\to -1}(x^2 - 2x + 1)$, b) $\displaystyle\lim_{x\to\infty}\frac{3x^2 - 7}{\sqrt{x^2+2}}$, c) $\displaystyle\lim_{x\to 0}\frac{\sin 3x}{2\sin 5x}$,

d) $\displaystyle\lim_{x\to\infty}\frac{3e^x + 4e^{-x}}{5e^x + 4e^{-x}}$, e) $\displaystyle\lim_{x\to 8^-}\frac{|x-8|}{x-8}$.

5. Find the following limits:

a) $\displaystyle\lim_{x\to 0^+}\sqrt{x}\ln x$, b) $\displaystyle\lim_{x\to 0^+} x^{\sin x}$, c) $\displaystyle\lim_{x\to\infty}\frac{(\ln x)^2}{x}$.

6. Find y' in each case. Do not simplify your answer.

a) $y = x^7 - 3x + 6 - \dfrac{1}{x^4}$, b) $y = e^{5x}\sin(x^2)$, c) $y = \dfrac{2\tan x}{\sqrt{1-x^2}}$,

d) $x^4y + 5y^6x^3 = 8$, e) $xe^{xy+3y} = y$, f) $y = (1+2x)^{1/x}$.

7. State the formal definition of the derivative of the function $f(x)$. Use the definition to calculate $f'(x)$ for $f(x) = \sqrt{3-5x}$.

8. Suppose $S(x) = \sqrt{x}$ for $x \geq 0$ and let f and g be differentiable functions about which the following is known:

$$f(3) = 2, \qquad f'(3) = 7, \qquad g(3) = 4, \qquad g'(3) = 5.$$

Compute the following:

$$(f+g)'(3)\ , \qquad (f\cdot g)'(3)\ , \qquad \left(\frac{f}{g}\right)'(3)\ , \qquad (S\circ g)'(3)\ , \qquad \left(\frac{f\cdot g}{f-g}\right)'(3)\ .$$

9. Suppose $f''(x) = -3x + \cos(\pi x)$ and $f(1) = 2$ and $f'(1) = -1$. What is $f(5)$?

10. A farmer with 450 feet of fencing wants to enclose the four sides of a rectangular region and then divide the region into four pens of equal size with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?

11. A ladder of length 15 feet is leaning against a wall when its base begins to slide along the floor, away from the wall. By the time the base is 12 feet away from the wall, the base is moving at a rate of 5 ft/sec. How fast is the top of the ladder sliding down the wall at that moment? How fast is the area of the triangle formed by the ladder, the wall, and the floor changing at that time?

12. Let $f(x) = \dfrac{3x}{x^2 - 1}$. Find the domain of the function, the intervals where $f(x)$ is increasing or decreasing, maximum and minimum, the concavity and inflection points, horizontal and vertical asymptotes of the graph of $f(x)$. Then sketch the graph of $f(x)$.

13. Same problem for $y = \dfrac{x^2}{x^2 + 3}$.

14. Sketch the graph of $f(x)$ which satisfies the following conditions:
$f'(1) = f'(-1) = 0$, $f'(x) < 0$ if $|x| < 1$, $f'(x) > 0$ if $1 < |x| < 2$, $f'(x) = -1$ if $|x| > 2$,
$f''(x) < 0$ if $-2 < x < 0$, inflection point $(0, 1)$.

15. Find the linearization of $f(x) = \sqrt{x+1}$ at $a = 15$ and use it to find an approximation to $\sqrt{15}$ and to $\sqrt{17}$. Use the second derivative to determine whether the estimate is greater or smaller than the actual value.

16. Find the derivatives of the following functions:
a) $F(x) = \displaystyle\int_{\pi}^{x} \tan(s^2)\, ds$, b) $g(x) = \displaystyle\int_{1}^{\cos x} \sqrt{1 - t^2}\, dt$.

17. Suppose that f is continuous on $[0, 4]$, $f(0) = 1$, and $2 \leq f'(x) \leq 5$ for all $x \in (0, 4)$. Show that $9 \leq f(4) \leq 21$.

18. Let $f(x) = 3x^2 - 2x^2 + x - 1$. Show that $f(x)$ must have a real root in $[0, 1]$.

19. Show that the equation $x^{101} + x^{51} + x - 1 = 0$ has exactly one real root.

20. Use two steps of Newton's method to approximate the root of the equation $x^4 + x - 4 = 0$ in the interval $[1, 2]$.

21. If a stone is thrown vertically upward from the surface of the moon with a velocity of 10m/s, its height (in meters) after t seconds is $h(t) = 10t - t^2$.
(a) What is the velocity of the stone after 3 seconds?
(b) What is the maximal height of the stone?

22. Find the absolute maximum and the absolute minimum of $f(x) = \dfrac{x}{x^2+1}$ in $[0, 2]$.

23. Find the most general antiderivative of the functions
a) $f(x) = 1 - x^3 + 5x^5 - 3x^7$, b) $g(x) = \dfrac{5 - 4x^3 + 3x^6}{x^6}$,
c) $h(x) = 3e^x + 7(\sec x)^2 + 5(1 - x^2)^{-\frac{1}{2}}$.

24. Evaluate the integral if it exists.
a) $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\, du$, b) $\displaystyle\int_0^2 y^2\sqrt{1 + y^3}\, dy$, c) $\displaystyle\int_1^5 \frac{dt}{(t-4)^2}$,
d) $\displaystyle\int_0^1 t^2 \cos(t^3)\, dt$, e) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$.

25. If $\displaystyle\int_1^5 f(x)\, dx = 12$ and $\displaystyle\int_4^5 f(x)\, dx = 3.6$ find $\displaystyle\int_1^4 f(x)\, dx$.

26. Find the area bounded by two curves:
a) $y_1 = 2x^2$ and $y_2 = 8x$, b) $y_1 = \sin(x)$ and $y_2 = \cos(x)$, $0 \le x \le \dfrac{\pi}{2}$.

27. Evaluate the limit $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sin\left(\frac{i\pi}{n}\right)\frac{\pi}{n}$.

28. a) Use the definition of the natural logarithm as an integral and compare areas to prove $\ln 2 < 1 < \ln 3$.
b) Use a) to deduce that $2 < e < 3$.

It was great to know you all. Thank you and all the best!