## Differentiation

**Definition:**

$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

**General formulas:**

*Product*: $(uv)' = u'v + uv'$

*Quotient*: $(u/v)' = [u'v - uv']/v^2$

*Chain rule*: $[f(u)]' = f'(u) \cdot u'$

*Constant multiple*: $(cu)' = cu'$

*Inverse*: $dx/dy = 1/(dy/dx)$

**Special functions:**

*Constants*: $c' = 0$

*Powers*: $\frac{d}{dx} x^n = nx^{n-1}$

*Exponential, logarithmic*:

$\frac{d}{dx} e^x = e^x$; $\quad \frac{d}{dx} \ln(x) = 1/x$

$\frac{d}{dx} a^x = \ln a \cdot a^x$

*Trigonometric*:

$\frac{d}{dx} \sin(x) = \cos(x)$; $\quad \frac{d}{dx} \cos(x) = -\sin(x)$

$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$; $\frac{d}{dx} \csc(x) = -\csc(x)\cot(x)$

$\frac{d}{dx} \tan(x) = \sec^2(x)$; $\frac{d}{dx} \cot(x) = -\csc^2(x)$

$\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$; $\frac{d}{dx} \cos^{-1}(x) = \frac{-1}{\sqrt{1-x^2}}$

$\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$; $\quad \frac{d}{dx} \cot^{-1}(x) = \frac{-1}{1+x^2}$

$\frac{d}{dx} \sec^{-1}(x) = \frac{1}{x\sqrt{x^2-1}}$

$\frac{d}{dx} \csc^{-1}(x) = \frac{-1}{x\sqrt{x^2-1}}$

**Mean Value Theorem:**

$f(x)$ cont. on $[a, b]$, diff. on $(a, b)$: one can solve

$$f'(x) = \frac{f(b) - f(a)}{b - a},$$

with $a < x < b$.

## Graphing

**Symmetry:**

*Even*: $f(-x) = f(x)$ (symmetric)

*Odd*: $f(-x) = -f(x)$ (skew symmetric)

**Asymptotes:**

*Horizontal*: $\lim_{x \to \pm\infty} f(x) = a$ (two cases)

*Vertical*: $\lim_{x \to a^\pm} f(x) = \pm\infty$ (four cases)

**Increasing/Decreasing:**

$f'$ *Positive on interval*: Increasing

$f'$ *Negative on interval*: Decreasing

**Local maxima/minima:**

*Critical numbers*: $f'(x) = 0$, or undefined;

*or Endpoints*

**Concavity, inflection:**

$f''(x) > 0$: upward; $\quad f''(x) < 0$: downward

$f''(x)$ *changes sign*: Inflection ($\sim$)

## Differentials and Newton's method

$dy = y'dx$; $y \approx y_0 + dy$

$f(a + \Delta x) \approx f(a) + f'(a)\Delta x$

*Newton's method*: $x^{\text{new}} = x - f(x)/f'(x)$ (iterate)

**Intermediate Value Theorem:**

If $f(x)$ is continuous on $[a, b]$, $f(a) < N < f(b)$, then the equation $f(x) = N$ is solvable, with $a < x < b$.

## Limits

*L'Hospital*: $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

– (when applicable)

$\lim_{n \to \infty} (1 + 1/n)^n = \lim_{h \to 0} (1 + h)^{1/h} = e$

**Squeeze Theorem:**

If $g_1(x) \le f(x) \le g_2(x)$ near $a$,
and $\lim_{x \to a} g_1(x) = \lim_{x \to a} g_2(x) = L$,
then $\lim_{x \to a} f(x) = L$.

*Example*: $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$

## Interpretations of derivatives

*1st*: Velocity or rate of change

*2nd*: Acceleration

*Logarithmic*: Relative rate of change

*Slope of tangent line*

## Integration

Integration gives the *signed area* between the curve and the $x$-axis (above−below).

**Fundamental Theorem of Calculus:**

($f$ *continuous*:) $\int_a^b f(x)dx$ is $F(b) - F(a)$, with $F(x)$ an antiderivative.

**Formulas:**

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \ (n \neq -1)$$

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

Read the **differentiation formulas** from right to left!

## Algebra

*Slope*: $\Delta y/\Delta x$; *Distance*: $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

*Quadratic formula*: $ax^2 + bx + c = 0$: then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$(a-b)(a+b) = a^2 - b^2$

$\ln(a^b) = b\ln(a)$

$\ln(ab) = \ln(a) + \ln(b)$; $\quad \ln(1/b) = -\ln(b)$

$\ln(1) = 0$; $\ln(e) = 1$

$\log_a(x) = \dfrac{\ln(x)}{\ln(a)}$

**Area:**

*Triangle*: 1/2 base×altitude

*Circle*: $\pi r^2$; *Sphere (surface)*: $4\pi r^2$

**Volume:**

*Box*: Product of dimensions.

*Sphere (inside)*: $\frac{4}{3}\pi r^3$

*Cylinder*: Base area × Height

*Cone*: $\frac{1}{3}$ Base area × Height

## Trigonometry

**Values:**

| $\theta$ | 0 | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| sin | 0 | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| cos | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | 0 |

**Right Triangles:**

*sine*: opposite/hypotenuse

*cosine*: adjacent/hypotenuse

*tangent*: opposite/adjacent

*secant*: 1/cosine

**Multiples:**

$\sin(2x) = 2\sin(x)\cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin^2(x/2) = \dfrac{1-\cos(x)}{2}$; $\cos^2(x/2) = \dfrac{1+\cos(x)}{2}$

**More identities:**

$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$

$\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$\tan(x+y) = \dfrac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

**Co-functions:**

$\cos(x) = \sin(\pi/2 - x)$; $\quad \cot(x) = \tan(\pi/2 - x)$

$\csc(x) = \sec(\pi/2 - x)$

## Numbers (*rough approximations*)

$\pi \approx 3.14 \quad e \approx 2.7 \quad \sqrt{2} \approx 1.4 \quad \sqrt{3} \approx 1.7$

$\ln 2 \approx .7 \quad \ln 10 \approx 2.3$