1. Prove that if S is a subset of 10 numbers from $\{1, \ldots, 100\}$ then there are two nonempty subsets A, B with $A \cap B = \emptyset$ such that the sum of the numbers in A is equal to the sum of the numbers in B

2. Prove that if S is any subset of 55 numbers chosen from $\{1, 2, \ldots, 100\}$ then there are two elements of S differing by exactly 9.

3. If a, b are integers, then $gcd(a, b)$ means the greatest common divisor of a and b. Prove that for any integers a and b there are integers s and t such that $sa + tb = gcd(a, b)$.

4. Let a, b, c, d be positive integers such that $ad - bc = 1$. Show that the fraction $(a + b)/(c + d)$ is in lowest terms.

5. Prove that some some positive multiple of 21 has 241 as its last 3 digits.

6. Prove that for any set of n integers, there is a subset of them whose sum is divisible by n.

7. Prove that if $2n + 1$ and $3n + 1$ are both perfect squares, then n is divisible by 40.

8. Prove that there are no integers x and y for which $x^2 + 3xy - 2y^2 = 122$.

9. Do there exist 1,000,000 consecutive integers such that each one contains a repeated prime factor?

10. Prove that for any integer n there is a multiple of n whose base 10 representation contains only 1's and 0's.

11. The Fibonacci sequence is the sequence $F_1, F_2, F_3, \ldots$ where $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Prove that if k is a divisor of some Fibonacci number then it is a divisor of infinitely many Fibonacci numbers.

12. How many 0's does 1000! end with?

13. Determine, as a function of the integer n, the number of ordered pairs (x, y) such that $1/x + 1/y = 1/n$.

14. Prove that not both integers $2^n - 1$ and $2^n + 1$ can be prime.

15. Find all pairs (m, n) of positive integers such that $|3^n - 2^n| = 1$

16. (a) Find all pairs (m, n) of positive integers such that $|3^n - 2^n| = 1$
    (b) Find all 4-tuples (p, q, m, n) where p, q are prime and m, n are positive integers that satisfy $|p^n - q^m| = 1$.

17. Prove that there are infinitely many natural numbers a with the property that $n^4 + a$ is not prime for any natural number n.