Formulas

$$\frac{d}{dx}\sin x = \cos x,\ \frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}a^x = a^x \ln a,\ \frac{d}{dx}x^n = nx^{n-1}$$

$$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$$

$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$$

$$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$$

The distance from $P(x_1, y_1, z_1)$ to $ax + by + cz + d = 0$ is $\dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

$$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$$

$$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{(\mathbf{a}\cdot\mathbf{b})}{(\mathbf{a}\cdot\mathbf{a})}\mathbf{a}$$

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$$

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$$

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \left|\frac{\mathbf{T}'(\mathbf{t})}{\mathbf{r}'(t)}\right| = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

$$\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{\mathbf{u}}{|\mathbf{u}|} \quad \text{where} \quad \mathbf{u} = \mathbf{r}'' - \frac{(\mathbf{r}'\cdot\mathbf{r}'')}{(\mathbf{r}'\cdot\mathbf{r}')}\mathbf{r}'$$

$$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$a_{\mathbf{N}} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$$

$$a_{\mathbf{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z},\ \frac{\partial z}{\partial y} = -\frac{F_y}{F_z},$$

$$\nabla f(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$$

$$\text{div}\,\mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z},\ \mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$$

$$\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}, \ \mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$$

$D = f_{xx}f_{yy} - f_{xy}^2$, $D > 0$ and $f_{xx} > 0$ implies local min
$D > 0$ and $f_{xx} < 0$ implies local max
$D < 0$ implies saddle

$\int\int_R f(x,y)\,dA = \int\int_S f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

Center of mass - 2 dimensions

$$mass = m = \int\int_D \rho(x,y)\,dA$$

$$M_y = \int\int_D x\,\rho(x,y)\,dA, \quad \bar{x} = \frac{M_y}{m}$$

$$M_x = \int\int_D y\,\rho(x,y)\,dA, \quad \bar{y} = \frac{M_x}{m}$$

Center of mass - 3 dimensions

$$mass = m = \int\int\int_E \rho(x,y,z)\,dV$$

$$M_{yz} = \int\int\int_E x\,\rho(x,y,z)\,dV, \quad \bar{x} = \frac{M_{yz}}{m}$$

$$M_{xz} = \int\int\int_E y\,\rho(x,y,z)\,dV, \quad \bar{y} = \frac{M_{xz}}{m}$$

$$M_{xy} = \int\int\int_E z\,\rho(x,y,z)\,dV, \quad \bar{z} = \frac{M_{xy}}{m}$$

Center of mass - a wire in the plane

$$mass = m = \int_C \rho(x,y)\,ds$$

$$M_y = \int_C x\,\rho(x,y)\,ds, \quad \bar{x} = \frac{M_y}{m}$$

$$M_x = \int_C y\,\rho(x,y)\,ds, \quad \bar{y} = \frac{M_x}{m}$$

Cylindrical coordinates:
$x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ dV = r\,drd\theta dz.$

Spherical coordinates:
$x = \rho\,\sin\phi\cos\theta,\ y = \rho\,\sin\phi\sin\theta,\ z = \rho\,\cos\phi,\ dV = \rho^2\sin\phi\,d\rho d\theta d\phi.$

Change of variables:
$$\int\int_R f(x,y)\,dx\,dy = \int\int_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

Surface Area:
$$A(S) = \int\int_D |\mathbf{r}_u \times \mathbf{r}_v|\,dA$$
$$A(S) = \int\int_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Green's Theorem:
$$\int_C P\,dx + Q\,dy = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA$$

Surface Integrals:
$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS = \int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int_D \mathbf{F}\cdot(\mathbf{r}_u\times\mathbf{r}_v)\,dA$$
$$d\mathbf{S} = (\mathbf{r}_u\times\mathbf{r}_v)\,dA, \quad dS = |\mathbf{r}_u\times\mathbf{r}_v|\,dA, \quad \mathbf{n} = \frac{\mathbf{r}_u\times\mathbf{r}_v}{|\mathbf{r}_u\times\mathbf{r}_v|}$$

Be careful with the next formula. It's only valid when the surface is the graph of $z = g(x,y)$.
$$\int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)\,dA$$

Stokes' Theorem:
$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int\int_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$$

The Divergence Theorem:
$$\int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int\int_E \operatorname{div}\mathbf{F}\,dV$$