Formulas

$\frac{d}{dx}\sin x = \cos x,\ \ \frac{d}{dx}\cos x = -\sin x$

$\frac{d}{dx}a^x = a^x \ln a,\ \ \frac{d}{dx}x^n = nx^{n-1}$

$\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta$

$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin\theta$

$\mathbf{r}(t) = \mathbf{r_0} + t\mathbf{v}$

The distance from $P(x_1, y_1, z_1)$ to $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2+b^2+c^2}}$.

$(z - z_0) = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$\text{comp}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$

$\text{proj}_{\mathbf{a}}\mathbf{b} = \frac{(\mathbf{a}\cdot\mathbf{b})}{(\mathbf{a}\cdot\mathbf{a})}\mathbf{a}$

$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c}$

$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\times\mathbf{b})\cdot\mathbf{c}$

$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \left|\frac{\mathbf{T'(t)}}{\mathbf{r}'(t)}\right| = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

$\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{\frac{3}{2}}}$

$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$

$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{\mathbf{u}}{|\mathbf{u}|} \quad \text{where} \quad \mathbf{u} = \mathbf{r}'' - \frac{(\mathbf{r}'\cdot\mathbf{r}'')}{(\mathbf{r}'\cdot\mathbf{r}')}\mathbf{r}'$

$\mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$

$a_{\mathbf{N}} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|}$

$a_{\mathbf{T}} = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|}$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z},\ \ \frac{\partial z}{\partial y} = -\frac{F_y}{F_z},$

$\nabla f(x, y, z) = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k}$

$\text{div}\,\mathbf{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z},\ \ \mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$

$$\operatorname{curl}\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix}, \ \mathbf{F} = P\,\mathbf{i} + Q\,\mathbf{j} + R\,\mathbf{k}$$

$$\begin{aligned} D = f_{xx}f_{yy} - f_{xy}^2,\ & D > 0 \ \text{ and } \ f_{xx} > 0 \text{ implies local min} \\ & D > 0 \ \text{ and } \ f_{xx} < 0 \text{ implies local max} \\ & D < 0 \ \text{ implies saddle} \end{aligned}$$

$\int\int_R f(x,y)\,dA = \int\int_S f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$

Center of mass - 2 dimensions

$$mass = m = \int\int_D \rho(x,y)\,dA$$

$$M_y = \int\int_D x\,\rho(x,y)\,dA, \quad \bar{x} = \frac{M_y}{m}$$

$$M_x = \int\int_D y\,\rho(x,y)\,dA, \quad \bar{y} = \frac{M_x}{m}$$

Center of mass - 3 dimensions

$$mass = m = \int\int\int_E \rho(x,y,z)\,dV$$

$$M_{yz} = \int\int\int_E x\,\rho(x,y,z)\,dV, \quad \bar{x} = \frac{M_{yz}}{m}$$

$$M_{xz} = \int\int\int_E y\,\rho(x,y,z)\,dV, \quad \bar{y} = \frac{M_{xz}}{m}$$

$$M_{xy} = \int\int\int_E z\,\rho(x,y,z)\,dV, \quad \bar{z} = \frac{M_{xy}}{m}$$

Center of mass - a wire in the plane

$$mass = m = \int_C \rho(x,y)\,ds$$

$$M_y = \int_C x\,\rho(x,y)\,ds, \quad \bar{x} = \frac{M_y}{m}$$

$$M_x = \int_C y\,\rho(x,y)\,ds, \quad \bar{y} = \frac{M_x}{m}$$

Cylindrical coordinates:
$x = r\cos\theta,\ y = r\sin\theta,\ z = z,\ dV = r\,dr d\theta dz.$

Spherical coordinates:
$x = \rho\,\sin\phi\cos\theta,\ y = \rho\,\sin\phi\sin\theta,\ z = \rho\,\cos\phi,\ dV = \rho^2\sin\phi\,d\rho d\theta d\phi.$

Change of variables:

$$\int\int_R f(x,y)\,dx\,dy = \int\int_S f(x(u,v),y(u,v))\left|\frac{\partial(x,y)}{\partial(u,v)}\right|\,du\,dv$$

Surface Area:

$$A(S) = \int\int_D |\mathbf{r}_u \times \mathbf{r}_v|\,dA$$

$$A(S) = \int\int_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

Green's Theorem:

$$\int_C P\,dx + Q\,dy = \int\int_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\,dA$$

Surface Integrals:

$$\int\int_S \mathbf{F}\cdot\mathbf{n}\,dS = \int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int_D \mathbf{F}\cdot(\mathbf{r}_u \times \mathbf{r}_v)\,dA$$

$$d\mathbf{S} = (\mathbf{r}_u \times \mathbf{r}_v)\,dA, \quad dS = |\mathbf{r}_u \times \mathbf{r}_v|\,dA, \quad \mathbf{n} = \frac{\mathbf{r}_u \times \mathbf{r}_v}{|\mathbf{r}_u \times \mathbf{r}_v|}$$

Be careful with the next formula. It's only valid when the surface is the graph of $z = g(x,y)$.

$$\int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right)\,dA$$

Stokes' Theorem:

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \int\int_S \operatorname{curl}\mathbf{F}\cdot d\mathbf{S}$$

The Divergence Theorem:

$$\int\int_S \mathbf{F}\cdot d\mathbf{S} = \int\int\int_E \operatorname{div}\mathbf{F}\,dV$$