

- (13) 1. Find an equation for the tangent line to the graph of  $y^2 = x^3 - 3xy + 3$  at the point  $(-2, 1)$ .

- (10) 2. Find equations for all vertical and horizontal asymptotes of the function

$$f(x) = \frac{3e^x + 5}{7e^x - 2}.$$

(All numbers used should be described by exact expressions, not decimal approximations. Thus you should write  $\sqrt{2}$ , not 1.414.)

- (15) 3. At a certain time, the length of a rectangle is 5 feet and its width is 3 feet. At that same moment, the length is *decreasing* at 0.5 feet per second and the width is *increasing* at 0.4 feet per second.

What is the length of the diagonal at that time?

How fast is the length of the diagonal changing? Is this length increasing or decreasing?

- (10) 4. Suppose that  $f(x) = \sqrt{2 + 7x^3}$ .

Compute  $f(1)$ .

Compute  $f'(1)$ .

Use the linearization (differential, tangent line approximation) of  $f$  at  $x = 1$  to estimate  $f(1.08)$ .

- (5) 5. A friend runs up to you and excitedly explains that she has found a function  $g$  with the following properties:

$g$  is continuous on  $[0, 1]$  and differentiable on  $(0, 1)$ .

$g(0) = 1$  and  $g(1) = 5$ .

$g'(x) \leq 3$  for all  $x$  in  $(0, 1)$ .

Explain why you doubt your friend's claim.

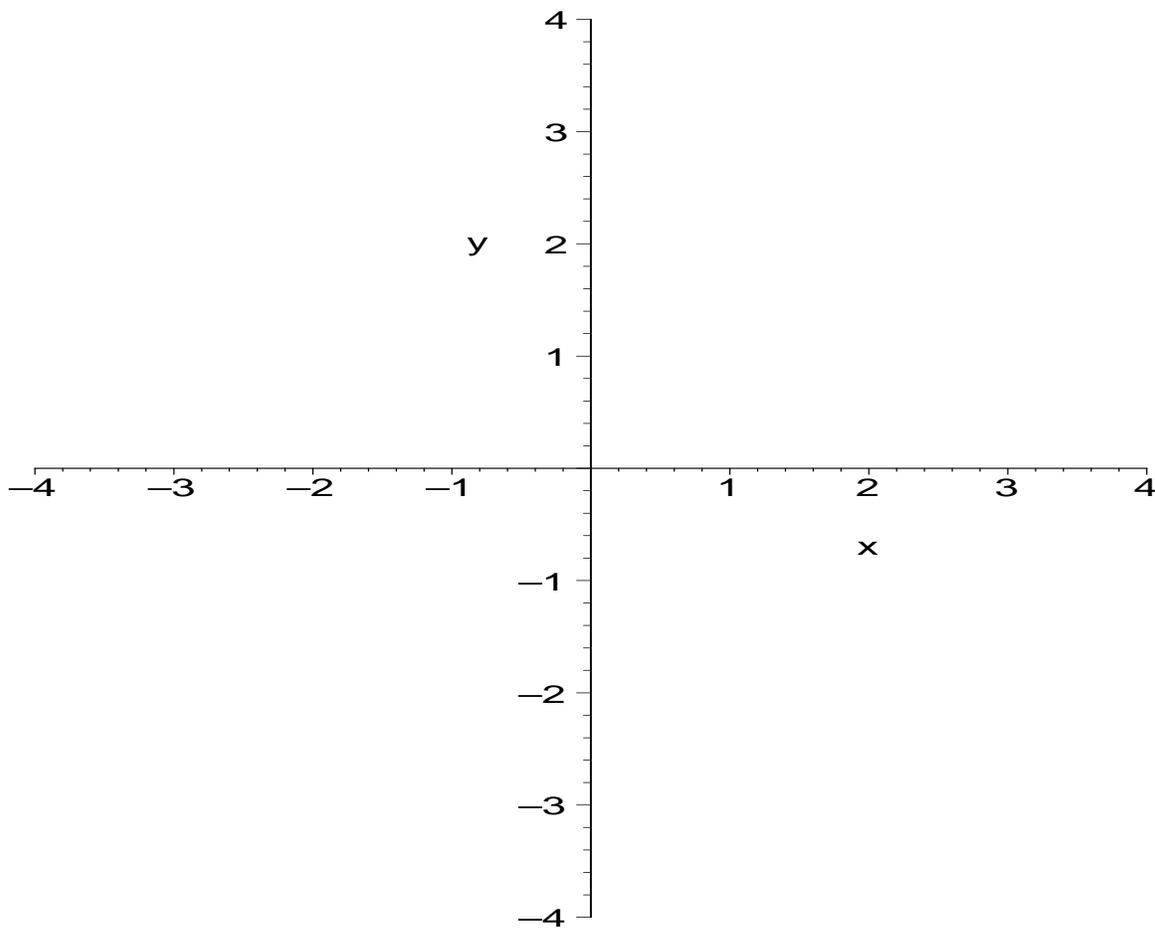
- (24) 6. Suppose that  $f(x) = \frac{x^2 + 3}{x^2 + x + 4}$ .

(a) What is the domain of  $f(x)$ ? Why?

(b) What are  $\lim_{x \rightarrow +\infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ ? Why?

(c) Use calculus to find all relative extreme values of  $f(x)$ .

- (d) The *range* of a function is the collection of all possible values of that function. What is the range of  $f$ ? Explain your answer carefully.
- (15) 7. You wish to build a shed in the shape of a rectangular box with a square floor. The materials for the walls cost \$1 per square foot and the materials for the floor and roof cost \$2 per square foot. You want the shed to have a volume of 250 cubic feet. What should the dimensions of the shed be in order to minimize the cost of materials?
- (8) 8. On the axes below sketch the graph of a function  $f$  with the following properties:  
The domain of  $f$  is  $(-4, 4)$  and  $f$  is differentiable at all points in its domain.  $f$  has a relative minimum at  $x = -2$  and a relative maximum at  $x = 2$ . At  $x = 0$  there is a horizontal tangent line and a point of inflection.



What is the total number of points of inflection of the function whose graph you have sketched?