1. (10 points) Find the equation of the tangent line to the curve with equation \( y = x^2 + 7 \) that is parallel to the line \( 2x - y - 3 = 0 \).

   **Equation of line is:** 

   \[
   \text{Slope of } 2x - y - 3 = 0 \text{ is 2. (} y = 2x - 3 \text{)}
   \]

   \[
   \frac{dy}{dx} = 2x \quad \text{slope of curve}
   \]

   \[
   \text{slope of curve = slope of line } \quad 2x = 2, \quad x = 1
   \]

   \[
   \text{Point on curve is (1,8) so } \quad (y-8) = 2(x-1)
   \]

2. If the area of a rectangle is 5 and its width is 3 less than twice its length, find the length of its diagonal.

   **Length of diagonal equals:** 

   \[
   \text{Area} = lw = 5
   \]

   \[
   w = 2l - 3
   \]

   \[
   l(2l - 3) = 5
   \]

   \[
   2l^2 - 3l - 5 = 0
   \]

   \[
   l = \frac{-3 \pm \sqrt{9 + 40}}{4} = \frac{3 \pm 7}{4}
   \]

   \[
   l = \frac{3 + 7}{4}, \quad w = 2\left(\frac{5}{2}\right) - 3 = 2
   \]

   \[
   \text{Diagonal} = \sqrt{(\frac{5}{2})^2 + 2^2}
   \]
3. Show that the equation \( \sqrt{x-1} = x^2 - 2 \) has at least one solution on the interval \((1, 2)\). Justify your answer.

\[
f(x) = \sqrt{x-1} - x^2 + 2
\]

Continuous because sum of polynomial and composition of polynomial + power function.

\[
f(c) = 0 \iff c \text{ satisfy equation above}
\]

\[
f(1) = 0 - 1 + 2 > 0
\]

\[
f(2) = 1 - 4 + 2 < 0
\]

By RTT, there is a \( c \) in \((1, 2)\) with \( f(c) = 0 \).

4. Find the derivatives. You do not have to simplify your answers once you have finished differentiating.

a. \( f(x) = \frac{\ln(x)}{1 + 5x^2} \)

\[
f'(x) = \frac{(1 + 5x^2) \cdot \frac{1}{x} - \ln(x) \cdot 10x}{(1 + 5x^2)^2}
\]

b. \( g(t) = \sin(t^2) \)

\[
g'(t) = (\cos(t^2))(2t)
\]
5. The half-life of a sample of radium is 1590 years. If the sample contains 100 grams on February 26, 2075, how much will be left on February 26, 2015?

<table>
<thead>
<tr>
<th>Amount of radium equals:</th>
</tr>
</thead>
</table>

\[
A(t) = 100e^{-\frac{\ln(2)}{1590}t}
\]

\[
A(1590) = 50 = 100 \cdot e^{-\frac{\ln(2)}{1590}}
\]

\[
\frac{1}{2} = e^{-\frac{\ln(2)}{1590}}
\]

\[
\ln\left(\frac{1}{2}\right) = -\frac{\ln(2)}{1590}
\]

\[
l = -\frac{\ln(2)}{1590}
\]

\[
A(1000) = 100 \cdot e^{-\frac{\ln(2)}{1590} \cdot 1000}
\]

\[
\frac{\ln(\frac{1}{2})}{1590}, 1000
\]

6. a. Use the algebraic techniques to find \( \lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} \).

Limit equals: \( \frac{1}{10} \)

\[
\lim_{x \to 25} \frac{\sqrt{x} - 5}{x - 25} = \lim_{x \to 25} \frac{\sqrt{x} + 5}{(\sqrt{x} - 5)(\sqrt{x} + 5)} = \lim_{x \to 25} \frac{1}{\sqrt{x} + 5} = \frac{1}{10}
\]

b. Find the derivative of \( g(x) = \frac{e^2}{1 - \sin(x)} \).

\[
g'(x) = \frac{e^2 \cos(x)}{(1 - \sin(x))^2}
\]

\[
g'(x) = \frac{(1 - \sin(x)) \cdot 0 - e^2 \cdot (-\cos(x))}{(1 - \sin(x))^2}
\]

\[
= \frac{e^2 \cos(x)}{(1 - \sin(x))^2}
\]
7a. Find \( \lim_{x \to 5} \frac{x - 5}{x^2 - 6x + 5} \). 

\[
\lim_{x \to 5} \frac{x - 5}{(x - 5)(x - 1)} = \lim_{x \to 5} \frac{1}{x - 1} = \frac{1}{4}
\]

b. For what values of \( c \) is the function 

\[
f(x) = \begin{cases} 
-2x + c & x \leq 3 \\
-4 & x > 3
\end{cases}
\]

continuous for all values of \( x \)?

\[
\lim_{x \to 3^-} f(x) = -2 \cdot 3 + c \\
\lim_{x \to 3^+} f(x) = \frac{-4}{3 - c}
\]

\( \lim f(x) \) exists when 

\[
\begin{align*}
-6 + c &= \frac{-4}{3 - c} \\
(-6 + c)(3 - c) &= -4 \\
-6c + 9c - 18 &= -4 \\
c^2 - 9c + 14 &= 0 \\
(c - 7)(c - 2) &= 0
\end{align*}
\]

\( c = 7 \) doesn't work because \( f \) is discontinuous at \( x = 7 \).
8. a. If \( y = \frac{x^4}{(\sqrt{x})^3} \), find \( y' \).

\[
y' = 3x^2, \ x \neq 0
\]

8b. If \( f(x) = x^3h(x) \) with \( h(-2) = 3 \) and \( h'(-2) = 4 \), calculate \( f'(-2) \).

\[
f'(x) = x^3h'(x) + 3x^2h(x)
\]

\[
f'(-2) = (2)^3 \cdot h'(-2) + 3(-2)^2 \cdot h(-2)
\]

\[
= -8 \cdot 4 + 3 \cdot 4 \cdot 3 = -32 + 36 = 4
\]
9. An astronaut standing on the edge of a cliff on the planet Pogo jumps directly upward and observes that exactly 4 seconds later she passes a point one foot above her initial position. Four seconds after that, she hits the ground at the base of the cliff. What is her initial velocity? How high is the cliff? \( h(t) = -\frac{1}{2}gt^2 + v_0t + s_0 \), and \( g = 2 \text{ ft/s}^2 \) near planet Pogo’s surface. Hint: What is \( h(4) \)? Plugging in gives you an equation that you can solve for \( v_0 \).

<table>
<thead>
<tr>
<th>Initial velocity:</th>
<th>( \frac{17}{4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cliff’s height is:</td>
<td>30</td>
</tr>
</tbody>
</table>

\[
h(t) = -\frac{1}{2}g t^2 + v_0 t + s_0 \quad \text{(from } g = 2 \text{)}
\]

\[
h(4) = h(0) + 1 \quad \text{From } \star
\]

\[
h(8) = 0 \quad \text{From } \star \star
\]

\[
-16 + 4v_0 = 1
\]

\[
v_0 = \frac{17}{4}
\]

\[
-32 + 8v_0 + s_0 = 0
-\frac{64}{4} + \frac{64}{4} + s_0 = 0
-64 + 34 + s_0 = 0
-30 + s_0 = 0
s_0 = 30
\]
10. The graph of a function $f(x)$ is given below

Find all values of $x$ in $[-5, 5]$ where $f$ fails to be

a. continuous.

$\chi = 1$

b. differentiable.

$\chi = -2, 1, 3$

c. For which values of $x$ is the derivative of $f$ equal to 0?

$\chi = 3$

d. Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.

$$\lim_{x \to 1^-} f(x) = -1$$

$$\lim_{x \to 1^+} f(x) = +1$$