Review questions for Chapter 11, Math 152

These review questions are **not** questions on the final exam with the numbers changed.

(1) Find a parametrization of the ellipse

\[
\frac{(x - a)^2}{A^2} + \frac{(y - b)^2}{B^2} = 1
\]

where \(a\) and \(b\) are constants and \(A, B\) are positive constants.

(2) Use a trigonometric identity to parametrize the curve

\[
y^2 - x^2 = 1 \quad y > 0
\]

(3) Consider the parametrization \(x = \cos t, y = \sin t\) of the circle \(x^2 + y^2 = 1\). Find the slope of the tangent to this circle at the point \((x, y) = (\sqrt{3}/2, 1/2)\) in two different ways: Using the parametrization and using Calculus I. Do you expect the same answer?

(4) One arch of a cycloid is given by

\[
x(t) = Rt - R \sin t \quad y(t) = R - R \cos t \quad \text{for} \quad 0 \leq t \leq 2\pi
\]

Find the area under this arch.

(5) Find the length of the cycloid arch in question (4).

(6) Consider the parametric curve

\[
C(t) = (R \cos t, -R \sin t) \quad \text{for} \quad \pi \leq t \leq 2\pi
\]

Use methods from Chapter 11 to find the surface area of the sphere generated by rotating this curve around the \(x\)-axis. You know what the answer should be.

See next page.
(7) Consider the parametric curve
\[ c(t) = (At, At^3) \] for \( 0 \leq t \leq 2 \)
where \( A > 0 \). We rotate this curve around the \( x \)-axis and get a surface. Find the surface area of this surface.

(8) Check the answer to question (7) using rectangular coordinates and a formula you learned earlier in this course.

(9) Find the center and radius of the circle that is given in polar form by
\[ r = \cos \theta \]

(10) Let \( d \) and \( \alpha \) be constants. Prove that the polar form
\[ r = d \sec(\theta - \alpha) \]
describes a line in the \( xy \)-plane.

(11) Use what we learned in section 11.4 to find the area of the region in the \( xy \)-plane that is under the circle \( x^2 + y^2 = 1 \) and over the line \( y = \sqrt{2}/2 \). You can check your answer by looking at the area between a circle and a square inscribed in the circle (and dividing by 4).

(12) Find the length of the cardioid
\[ r = 1 - \cos \theta \quad 0 \leq \theta \leq 2\pi \]