Review Sheet for Exam 3, Math 152

These review questions are not the questions on the exam with the numbers changed.

(1) A student used the following exact words when asking a question: “I see 1/n as the general term in a question on series. Is this a case of convergence or a case of divergence?” The professor answered, “It depends on what you are asking. Are you talking about a sequence or a series with general term 1/n? Explain in a few words why the question of convergence/divergence of a sequence with general term 1/n is different from the question of convergence/divergence of a series with general term 1/n.

(2) Assume that we have functions $f(x)$ and $g(x)$ which have Taylor series centered at 1 that begin in the following way: $f(x) = -3 + 7(x - 1) + 5(x - 1)^2 + (-4)(x - 1)^3 + \cdots$, $g(x) = 8 + 3(x - 1) + (-2)(x - 1)^2 + 7(x - 1)^3 + \cdots$. Find the first four nonzero terms of the Taylor series centered at 1 of the product $f(x)g(x)$.

(3) Find the first three nonzero terms of the Maclaurin series of $\tan x$.

(4) Find the Maclaurin series for $\frac{1}{4 + x^2}$.

(5) Assume that the Taylor series centered at 5 of the function $e^{3x}$ is $\sum_{n=0}^{\infty} a_n(x - 5)^n$. Find $c$ and find a formula for $a_n$.

(6) Evaluate $\lim_{n \to \infty} \left( \ln(3n + 4) - \ln(2n + 7) \right)$.

(7) Evaluate $\lim_{n \to \infty} \left( \frac{n}{n + 1} \right)^n$.

(8) Consider the series $\sum_{n=1}^{\infty} \frac{n^5}{2^n}$. Which of the following tests for convergence/divergence is appropriate for this series?
   - The $n$th Term Divergence Test
   - The Ratio Test
   - The Root Test
   - The Integral Test

Determine convergence/divergence using one of these tests.

(9) Determine convergence or divergence of the series $\sum_{n=3}^{\infty} \frac{\sin(e^n + \sqrt{n})}{n^2 + n}$.
(10) Consider the series \( \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \). How many terms do we need to add up in order to guarantee that the resulting partial sum approximates the sum of the infinite series with accuracy better than \( \frac{1}{100} \)?

(11) Consider the power series \( \sum_{n=0}^{\infty} \frac{1}{(n+7)^2} (x+10)^n \). Find the interval of convergence of this power series.

(12) Assume that we have an interval \( S \) with endpoints \( a \) and \( b \), which are real numbers. Assume \( a < b \). We are not specifying whether the interval is open or closed at these endpoints. We do tell you that there is a certain power series \( \sum_{n=0}^{\infty} a_n (x - c)^n \) with the following property: \( S \) equals (exactly) the set of points \( x \) where this power series converges. We also tell you that the length of the interval \( S \) is equal to 8 (exactly) and that the power series converges conditionally at \( x = 7 \). What can you say about \( a \) and \( b \)? What can you say about the radius of convergence? What can you say about the center of the power series?

(13) Find the sum of the telescoping series \( \sum_{n=7}^{\infty} \frac{2n + 1}{n^2 (n+1)^2} \).

(14) Determine convergence or divergence of the series \( \sum_{n=3}^{\infty} \frac{n^2 - n - 1}{n^3 + 2n + 1} \).

(15) Which of the following statements is true?
(a) If a series has one or more ! in it, then we should always use the Ratio Test to determine convergence or divergence.
(b) If a series has one or more ! in it, then the Ratio Test is likely the best test for convergence or divergence.
(c) If a series has one or more ! in it, then the Root Test is just as easy to apply as the Ratio Test.