

**Math 170S**  
**Homework for Section 6.7** \*†  
Instructor: Swee Hong Chan

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**Note:** Homework will not be collected, but the question for the quizzes might be picked from the homework questions.

1. Solve Problem 6.7-4.
2. Let  $X$  be the gamma distribution with the fixed parameter  $\alpha$  and unknown parameter  $\theta$ , i.e.,

$$f_{\theta}(x) = \frac{1}{\Gamma(\alpha)\theta^{\alpha}} x^{\alpha-1} e^{-x/\theta} \quad x \geq 0,$$

where  $\Gamma$  is the gamma function.

- (a) Find a sufficient statistic for  $\theta$ .
  - (b) Give examples of two more different sufficient statistics for  $\theta$ .
3. Let  $X$  be an independent random variables indexed by an unknown parameter  $\theta$  with pdf

$$f_{\theta}(x) = \begin{cases} \exp(K(x)p(\theta) + S(x) + q(\theta)) & \text{if } x \in A; \\ 0 & \text{otherwise;} \end{cases}$$

where  $K(x)$ ,  $p(\theta)$ ,  $S(x)$ ,  $q(\theta)$  are some given functions, and  $A$  a given subset of  $\mathbb{R}$ .

- Deduce that  $\sum_{i=1}^n K(x_i)$  is a sufficient statistics for  $\theta$ .
- Derive sufficient statistics for parameters of Bernoulli( $p$ ), Exp ( $\lambda$ ), and Poisson( $\lambda$ ) using part (i).

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†This homework is based on Hanbaek Lyu's and Liza Rebrova's homeworks from the previous quarter, and I would like to thank her for her generosity here. "*Nanos gigantum humeris insidentes* (I am but a dwarf standing on the shoulders of giants)".