Math 170S
Homework for Section 6.5 [*][†]
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Note: Homework will not be collected, but the question for the quizzes might be picked from the homework questions.

1. Solve Problem 6.5-4.

2. Let $x_1, \ldots, x_n$ be (fixed real numbers), let $\epsilon_1, \ldots, \epsilon_n$ be independent normal random variables with mean 0 and variance σ^2. Let $y_1, \ldots, y_n$ be given by

$$y_i := \alpha + \beta x_i + \epsilon_i,$$

where α and β are given fixed constants. Let $\widehat{\alpha}$, $\widehat{\beta}$, and $\widehat{\sigma}^2$ be the MLEs for the linear regression, i.e., $\widehat{\alpha}$, $\widehat{\beta}$ is given the formula in Theorem 2 of the lecture notes.

- Show that

$$E[\widehat{\alpha}] = \alpha; \qquad Var[\widehat{\alpha}] = \frac{\sigma^2}{n} \left(\frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n (x_i - \overline{x})^2} \right).$$

- Show that

$$E[\widehat{\beta}] = \beta; \qquad Var[\widehat{\beta}] = \frac{\sigma^2}{\sum_{i=1}^n (x_i - \overline{x})^2}.$$