

Math 170S
Homework for Section 6.5 *†
Instructor: Swee Hong Chan

Note: Homework will not be collected, but the question for the quizzes might be picked from the homework questions.

1. Solve Problem 6.5-4.
2. Let x_1, \dots, x_n be (fixed real numbers), let $\epsilon_1, \dots, \epsilon_n$ be independent normal random variables with mean 0 and variance σ^2 . Let y_1, \dots, y_n be given by

$$y_i := \alpha + \beta x_i + \epsilon_i,$$

where α and β are given fixed constants. Let $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\sigma}^2$ be the MLEs for the linear regression, i.e., $\hat{\alpha}$, $\hat{\beta}$ is given the formula in Theorem 2 of the lecture notes.

- Show that

$$E[\hat{\alpha}] = \alpha; \quad \text{Var}[\hat{\alpha}] = \frac{\sigma^2}{n} \left(\frac{\sum_{i=1}^n x_i^2}{\sum_{i=1}^n (x_i - \bar{x})^2} \right).$$

- Show that

$$E[\hat{\beta}] = \beta; \quad \text{Var}[\hat{\beta}] = \frac{\sigma^2}{\sum_{i=1}^n (x_i - \bar{x})^2}.$$

*Version date: Thursday 23rd April, 2020, 21:33.

†This homework is based on Hanbaek Lyu's and Liza Rebrova's homeworks from the previous quarter, and I would like to thank her for her generosity here. “*Nanos gigantum humeris insidentes* (I am but a dwarf standing on the shoulders of giants)”.