Construction of $(1+\epsilon,\beta)$-spanners in general graphs

Michael Elkin, Weizmann Institute, Rehovot, Israel

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Abstract:

An (α,β)-spanner of a graph G is a subgraph H such that $d_H(u,w) \leq \alpha \cdot d_G(u,w) + \beta$ for every pair of vertices u, w, where $d_{G'}(u,w)$ denotes the distance between two vertices u and v in G'. It is known that every graph G has a polynomially constructible $(2\kappa-1,0)$-spanner (a.k.a. *multiplicative* $(2\kappa-1)$-spanner) of size $O(n^{1+1/\kappa})$ for every integer $\kappa \geq 1$, and a polynomially constructible $(1,2)$-spanner (a.k.a. *additive* 2-spanner) of size $O(n^{3/2})$. This paper explores hybrid spanner constructions (involving both multiplicative and additive factors) for general graphs and shows that the multiplicative factor can be made *arbitrarily close to 1* while keeping the spanner size arbitrarily close to $O(n)$, at the cost of allowing the additive term to be a sufficiently large constant. More formally, we show that for any constant $\epsilon, \delta > 0$ there exists a constant $\beta = \beta(\epsilon,\delta)$ such that for every n-vertex graph G there is an efficiently constructible $(1+\epsilon,\beta)$-spanner of size $O(n^{1+\delta})$. It follows that for any constant $\epsilon, \delta > 0$ there exists a constant $\beta(\epsilon,\delta)$ such that for any n-vertex graph $G = (V,E)$ there exists an efficiently constructible subgraph (V,H) with $O(n^{1+\delta})$ edges such that $d_H(u,w) \leq (1+\epsilon)d_G(u,w)$ for every pair of vertices $u, w \in V$ such that $d_G(u,w) \geq \beta(\epsilon,\delta)$. The talk is based on a joint paper with David Peleg, that appeared at STOC 2001.