

As explained on the course web site, when handing in homework, it is not enough to provide just the final answer to a computational problem. You must provide a clear and brief explanation of how the answer is obtained. Here are a few examples of acceptable solutions for problems from chapter 1.

- Chapter 1, Problem 19a. (a) Since any 3 of the women can be on the committee there are  $\binom{8}{3}$  ways to choose the women on the committee. Let A and B denote the two men who won't served on the committee together. There are  $\binom{6}{3}$  ways to choose 3 men out of 6. However, we must exclude those groups of 3 that contain both A and B, of which there are 4, so there are  $\binom{6}{3} - 4$  ways to choose the men on the committee. By the basic principle of counting (from the book) there are  $\binom{8}{3} \times (\binom{6}{3} - 4)$  ways to choose the committee.
- Chapter 1, Problem 19c. Now let A denote the man and B the woman who won't serve together. If we ignore this restriction there are  $\binom{8}{3}$  ways to choose the women on the committee and  $\binom{6}{3}$  ways to choose the men on the committee, so by the basic principle of counting there are  $\binom{8}{3}\binom{6}{3}$  ways to choose the committee (ignoring the restriction). From this we must subtract off the committees that contain both A and B. If we choose man A and woman B, then there are  $\binom{5}{2}$  ways to choose the remaining men for the committee and  $\binom{7}{2}$  ways to choose the remaining women for the committee so there are  $\binom{5}{2}\binom{7}{2}$  ways to choose a committee containing A and B. Therefore there are  $\binom{8}{3}\binom{6}{3} - \binom{7}{2}\binom{5}{2}$  ways to choose a committee that does not have both A and B.
- Chapter 1, Problem 22. In the figure, to get from point A to point B passing through the circled point, using the basic principle of counting, we multiply the number of ways to get from A to the circled point by the number of ways to get from the circled point to B. To get from A to the circled point we need to take four steps of which two are "up" steps and two are "right" steps. The up steps can be any 2 of the 4 steps, so there are  $\binom{4}{2}$  ways to get from A to the circled point Similarly, to get from the circled point to point B we must take 3 steps of which one is an "up" step and two are "right" steps so there are  $\binom{3}{1}$  ways. Overall there are  $\binom{4}{2}\binom{3}{1}$  ways to get from A to B through the circled point.
- Chapter 1, Theoretical exercise 3. In how many ways can  $r$  objects be selected from a set of  $n$  objects if the order of selection is considered relevant? **Solution 1.** We must choose a sequence  $a_1, \dots, a_r$  of distinct objects from the  $n$  objects. We can choose  $a_1$  in  $n$  ways. Having chosen  $a_1$  we can choose  $a_2$  in  $n - 1$  ways. Having chosen  $a_1, a_2$  we can choose  $a_3$  in  $n - 2$  ways. Continuing in this way, and using the basic principle counting, we can choose  $a_1, a_2, \dots, a_r$  in  $n \times (n - 1) \times (n - 2) \times \dots \times (n - r + 1)$  ways. **Solution 2.** We can first choose the set of objects (ignoring the order) in  $\binom{n}{r}$  ways. Then we can order the objects in  $r!$  ways. By the basic principle of counting there are  $\binom{n}{r}r!$  ways to choose an ordered list of  $r$  out of  $n$ .

[NOTE: The answers given by solutions 1 and 2 are the same; check this!]

- Chapter 1, Theoretical exercise 11. We are supposed to explain why for any positive integers  $n, k$  with  $n \geq k$ , we have:  $\binom{n}{k} = \sum_{i=k}^n \binom{i-1}{k-1}$ . **Solution.**  $\binom{n}{k}$  counts the number of subsets of  $\{1, \dots, n\}$  of size exactly  $k$ . Let us classify these subsets according to their largest element. The largest element is one of the integers between  $k$  and  $n$ . If the largest element is  $i$  then the  $k - 1$  remaining elements are chosen from the set  $\{1, \dots, i - 1\}$  and so there are  $\binom{i-1}{k-1}$  ways to do this. Summing over  $i$  between  $k$  and  $n$  gives the left hand side of the equality.