Mathematical Theory of Probability(640:477:03)
Fall 2013
Assignment 9 [1]

- The homework requirements given on the course web page were revised last week to **add the requirement of a cover page** in a certain format, to help ensure that the grader grades all of your work. Please read this and follow it.
- The practice problems are from the NINTH edition of the course text "A first course in probability".

Problems for practice (not to be handed in): Problems 6.24, 6.25, 6.29, 6.31, 6.34, 6.38, 6.44. 7.1, 7.2, 7.4, 7.7,7.8.7.9

Problems to be handed in.

(1) The joint density of X, Y is given by $f(x,y) = \frac{x}{2} + \frac{y}{4}$ for $0 \leq x \leq 1$ and $0 \leq y \leq 2$.
   (a) Determine the marginal distribution of X.
   (b) Determine the conditional distribution of X given $Y = 1/2$.

(2) Let X_1, X_2, X_3 be independent discrete random variables that are each uniform over the set $\{1, 2, 3\}$.
   (a) Determine the probability mass function of $S = X_1 + X_2 + X_3$.
   (b) Determine the joint probability mass function of (X_1, X_2, X_3) given that $S = 6$.
   (c) Determine the marginal probability mass function of X_1 conditioned on $S = 6$.

(3) Let $X_1, X_2, \ldots, X_{10}$ be independent random variables each having a normal distribution with mean 10 and variance 20.
   (a) Estimate the probability that $X_1 \geq 12$.
   (b) Let Y be the average of X_1 and X_2. Estimate the probability that $Y \geq 12$.
   (c) Let Z be the average of $X_1, \ldots, X_{10}$. Estimate the probability that $Z \geq 12$.
   (d) (Not to be graded). The above results illustrate a general principle involving averages of identical normal random variables. Try to formulate such a principle.

(4) A particular rare disease affects men at a different rate than women. In a given year, the probability that a given man contracts the disease is $1/600,000$ while the probability that a given woman contracts the disease is $1/400,000$. For a city with 1,000,000 men and 1,000,000 women, estimate the probability that there are more than 6 cases of the disease.

(5) Suppose X and Y have joint density function $f(x,y)$ which is given by $\frac{6}{x^4 y^3}$ when $x \geq 1$ and $y \geq 1$ and is 0 otherwise.
   (a) Suppose Z is the maximum of X and Y. Find the Cumulative distribution function for Z.
   (b) (Not to hand in.) Find the expected value of Z.

[1] Version: 11/26/13