

Mathematical Theory of Probability(640:477:03)
Fall 2013
Assignment 7 ¹

- The homework requirements given on the course web page were revised last week to **add the requirement of a cover page** in a certain format, to help ensure that the grader grades all of your work. Please read this and follow it.
- The practice problems are from the NINTH edition of the course text “A first course in probability”.

Problems for practice (not to be handed in): Problems 5.24, 5.26, 5.29, 5.31, 5.34, 5.39, 5.42

Problems to be handed in.

- (1) The following problem makes use of the table on page 190 of the book. Suppose that X is a Normal random variable with mean 5 and variance 10.
 - (a) Estimate the probability that X is between 4 and 7.
 - (b) Estimate the endpoints of an interval I that is symmetric around the mean and is as small as possible such that $P[X \in I] \geq .8$.
- (2) Suppose X is an exponential random variable with mean 3. Suppose $Y = e^X$. Find the pdf for Y .
- (3) If we roll a fair die 1000 times.
 - (a) Use the normal approximation (and the table on page 190 of the book) to estimate the probability that 6 comes up at least 200 times.
 - (b) Approximately how many times do we need to roll the die in order that the following holds: The probability that the number of 6's rolled exceeds the expected number of 6's by more than 5% is less than 1/20.
- (4) The distribution of the lifetime of a particular manufactured product has hazard rate function $\lambda(t) = t^3/9$ for $t > 0$. Determine:
 - (a) The probability that the item survives for at least 3 years.
 - (b) The probability that the item survives for between 2 and 3 years.
 - (c) The conditional probability that the item survives for at least 3 years given that it survives at least 2 years.
- (5) Suppose we have a polynomial function $p(x) = x^2 - Ax + 2A - 3$, where A is an unspecified constant. Suppose that A is chosen uniformly at random between 0 and 10. What is the probability that the function $p(x)$ is strictly positive for all x ? (Hint: First determine conditions on the constants c and d such that the polynomial $x^2 + cx + d$ is positive for all x .)

¹Version: 10/30/13