Mathematical Theory of Probability(640:477:03)
Fall 2013
Assignment 6 [1]

- The homework requirements given on the course web page were revised last week to **add the requirement of a cover page** in a certain format, to help ensure that the grader grades all of your work. Please read this and follow it.
- The practice problems are from the NINTH edition of the course text "A first course in probability".

Problems for practice (not to be handed in): **Chapter 5:** Problems 5.2, 5.4, 5.8, 5.13, 5.16, 5.18, 5.23, 5.25, 5.28, 5.32

Problems to be handed in.

(1) Suppose that X is a random variable whose density is given by $f(x) = 0$ for $x < 0$, $f(x) = Ce^{-2x}$ if $0 \leq x \leq 2$ and $f(x) = Ce^{-(x+2)}$ if $x > 2$, where C is some constant.
  (a) Determine C.
  (b) Find the CDF for X.

(2) (This problem counts as two usual problems (10 points)) Let X be a continuous random variable whose probability density function has the form:

$$f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{Bx}{A} & \text{if } 0 \leq x \leq A \\ B & \text{if } A < x \leq A + C \\ 0 & \text{if } x > A + C \end{cases}$$

where A, B, C are undetermined positive constants.
  (a) Sketch the graph of the density function. Show the shape of the graph and label the (x, y) coordinates of key points (these labels will depend on A, B, C).
  (b) Use the fact that the definite integral of the PDF is 1 to determine an equation relating A, B, C.
  (c) Express the expected value of X in terms of A, B, C.
  (d) Express the probability that $X \geq A + C/2$ in terms of A, B, C.
  (e) Given that $E[X] = 71/24$ and $Prob[X \geq A + C/2] = 3/8$ determine A, B and C.

(3) Suppose we choose a ray starting at the origin which makes an angle θ with the positive real axis, where θ is chosen uniformly between 0 and π. Consider the point (X, Y) where the ray hits the unit circle.
  (a) Find $E[X]$.
  (b) Find $E[Y]$.
  (c) Find $Var(X)$.

(4) Suppose that X and Y are independent random variables both having pdf $f(x) = e^{-x}$ for $x \geq 0$ and $f(x) = 0$ for $x < 0$.
  (a) Let Z be the maximum of X and Y. Find the pdf for Z. (Hint: Notice that for any number t, $P[Z \leq t] = P[(X \leq t) \text{ AND} (Y \leq t)]$. Use this and the fact that X and Y are independent to find the CDF of Z.
  (b) Let W be the minimum of X and Y. Find the pdf of W.

[1]Version: 10/30/13