

Mathematical Theory of Probability(640:477:03)
Fall 2013
Assignment 4 ¹

- The homework requirements given on the course web page were revised last week to **add the requirement of a cover page** in a certain format, to help ensure that the grader grades all of your work. Please read this and follow it.
- Problems are taken from the NINTH edition of the course text “A first course in probability”.
- “Problems” for practice (not to be handed in): **Chapter 4:** 4.4, 4.14, 4.15, 4.19, 4.23, 4.32, 4.38, 4.41, 4.44

Problems to be handed in.

- (1) A student takes a multiple choice test with 5 questions. Each question has 4 possible answers. The test is scored by giving 3 points for a correct answer and -1 for an incorrect answer. Suppose that for each question, a given student is able to eliminate one obviously incorrect answer, and then randomly guesses one of the remaining answers. (a) Find the probability mass function for the student’s score. (b) Compute the expected value (c) Compute the variance of the student’s score. (There is a simpler way to compute the expected value and variance than just applying the definition; see if you can find it.)
- (2) A device is built from four components, each of which fails independently with probability p . The device functions properly provided that at least 3 of the components are working. (a) If 8 devices are built, what is the probability that at least 6 function properly? (b) What is the expected number of components that work properly?
- (3) We have three coins. One has probability of heads equal to $1/4$, one has probability of heads equal to $3/4$ and one is a fair coin. We select a coin at random, and flip it four times. If the number of heads is two, find the conditional probability that we selected the fair coin. Express your answer as a fraction in lowest terms.
- (4) We have a coin whose probability of heads is p . Suppose we flip the coin until the number of heads seen is exactly 10. Let Y be the total number of coin flips. Find the probability mass function for Y . (Hint: notice that when we stop, the last coin flip must be heads.)
- (5) Suppose we have an urn with w white balls and r red balls. We select k balls without replacement from the urn (where k is a number less than or equal to $w + r$). Let X be the random variable which is the number of white balls in the sample. Find the probability mass function for X . (Hint: You may want to first try picking specific numbers for w , r and k and solving the problem for those numbers, and then generalize your solution to work for arbitrary w, r and k).

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