Intro to Mathematical Reasoning (Math 300) – Homework 9[1]

1. For each of the following functions, determine if the function is one-to-one, onto (the given target set), or both. Prove your answers.
   (a) Let $f : \mathbb{Z} \longrightarrow \mathbb{Z}$ be given by the rule $f(n) = n + 12$ for all $n \in \mathbb{Z}$.
   (b) Let $g : \mathbb{N} \longrightarrow \mathbb{N}$ be given by the rule $f(n) = n + 12$ for all $n \in \mathbb{N}$.
   (c) Let $h : \mathbb{Z} \longrightarrow \mathbb{Z}$ be given by the rule $h(x) = x/2$ for x even and $h(x) = (x-1)/2$ for x odd.
2. Let $r : \mathbb{N} \times \mathbb{N} \longrightarrow \mathbb{N}$ be given by the rule $r(a, b) = 2^{a-1}(2b - 1)$. Prove that r is one-to-one and onto. (Hint: To prove that r is one-to-one let (a_1, b_1) and (a_2, b_2) be elements of $\mathbb{N} \times \mathbb{N}$ and assume $f(a_1, b_1) = f(a_2, b_2)$. Use the fact that a number can't be both odd and even to prove that $a_1 = a_2$ and then prove that $b_1 = b_2$. To prove that r is onto, use complete induction to prove that for all natural numbers n, there are natural numbers a and b such that $n = 2^{a-1}(2b - 1)$. The basis case for the induction should be the case that n is odd.)
3. End of chapter 5, problem 5 (a) and (b).
4. End of chapter 5, problem 9.
5. End of chapter 5, problem 24.
6. Prove Theorem 8.2.4.
7. Problem 8.3.8.

[1]Version 3/29/04