Intro to Mathematical Reasoning (Math 300) – Homework 6[1]

1. (a) Prove that every positive integer can be written as the sum of one or more distinct powers of 2. (Hint: Use the principle of complete induction. For the induction step, divide into cases according to whether n is a power of 2 or not. (Here we say that a number is a power of 2 if it is equal to 2^k for some nonnegative integer k). In the case that n is not a power of 2, let m be the largest power of 2 that is less than or equal to n. Apply the induction hypothesis to $n - m$.)

   (b) It is not true that every positive integer can be written as the sum of one or more distinct powers of 3. Therefore your argument in the previous problem should not work if you modify the argument by replacing powers of 2 by powers of 3. (If it *does* work for powers of 3, then there is something wrong with your proof! Find the error and correct it before doing this problem.) Explain precisely where your argument fails if you replace powers of 2 by powers of 3.

2. (a) Give an example of a relation on $\{a, b, c, d\}$ that is symmetric and reflexive but not transitive.

   (b) Give an example of a relation on $\{a, b, c, d\}$ that is reflexive and transitive but not symmetric.

   (c) Give an example of a relation on $\{a, b, c, d\}$ that is symmetric and transitive but not reflexive.

   (d) Is it possible to have a relation on $\{a, b, c, d\}$ that is neither symmetric nor antisymmetric? Explain.

   (e) Is it possible to have a relation on $\{a, b, c, d\}$ that is both symmetric and antisymmetric? Explain.

3. Let Q be the relation defined on $\mathbb{R}$ consisting of all ordered pairs (x, y) such that $x < y + 1$. For each of the four properties in definition 4.1.8 determine whether Q satisfies the property. In each case, carefully prove your answer.

4. Do exercise 4.2.17 and prove Theorem 4.2.18 in the book

5. Prove Theorem 4.2.22 in the book.

[1] Version 2/22/04