Intro to Mathematical Reasoning (Math 300) – Homework 5[1]

1. Textbook, problem 3.2.3.

2. Textbook, problem 3.2.6.

3. Prove: For all real numbers $x \geq -1$ and for all nonnegative integers n, $(1+x)^n \geq 1 + nx$.

4. Prove that for all nonnegative integers n, $\sum_{i=1}^{2^n} \frac{1}{i} \geq 1 + \frac{n}{2}$. (It is strongly recommended that you test this out for some small values of n before trying to prove it. Also, in your proof you may use the following lemma without proof: For all positive integers k, and all lists $b_1, \ldots, b_k$ of real numbers satisfying $b_1 \geq b_2 \geq \cdots \geq b_{k-1} \geq b_k$, we have $\sum_{j=1}^{k} b_j \geq k b_k$.)

5. Let a, b be real numbers. Let $c_1, c_2, c_3, \ldots$ be an infinite sequence of real numbers such that for each positive integer k, $c_k = ak - b$. For each positive integer n, let $s_n = \sum_{k=1}^{n} c_k$. Prove that for each positive integer n, $s_n = \frac{a}{2}n^2 + (\frac{a}{2} - b)n$.

6. Let $a_0, a_1, a_2, \ldots$ be an infinite sequence of real numbers such that for $n \geq 3$, $a_n = a_{n-1} + 2a_{n-2}$. Prove that for each $n \geq 0$, if n is even then $a_n = \frac{1}{3}(2^n(a_0 + a_1) + 2a_0 - a_1)$ and if n is odd then $a_n = \frac{1}{3}(2^n(a_0 + a_1) + a_1 - 2a_0)$.

7. Prove: For any integer n and for any list $A_1, \ldots, A_n$ where each A_i is a set, if for all $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, n\}$, we have $A_i \subseteq A_j$ or $A_j \subseteq A_i$ then then there is a set $k \in \{1, 2, \ldots, n\}$ such that $A_k \subseteq A_i$ for all $i \in \{1, 2, \ldots, n\}$.

[1] Version 2/16/04