

1. Textbook: exercise 2.4.4
2. Recall the following definition: For any two sets A and B , the *difference set* $A \setminus B$ is the set consisting of those objects that are members of A but not members of B .
 - (a) Prove or disprove: For all sets A, B, C , if $A \setminus C = B \setminus C$ then $A = B$.
 - (b) Prove or disprove: For all sets A, B, C , if $A \setminus C = B \setminus C$ and $C \setminus A = C \setminus B$ then $A = B$.
3. Prove or disprove: For any three sets X, Y, Z , if $X \cap Y \neq \emptyset$, $X \cap Z \neq \emptyset$ and $Y \cap Z \neq \emptyset$ then $X \cap Y \cap Z \neq \emptyset$.
4. Textbook: Prove Theorem 2.5.5
5. Textbook: end of chapter 2, problem 4.
6. If S is a set, a *partition* of S is a set $\mathcal{P}$ of subsets of S satisfying the following three conditions: (i) $\emptyset \notin \mathcal{P}$, (ii) for each $s \in S$, there is an $M \in \mathcal{P}$ such that $s \in M$, and (iii) For each pair $M_1, M_2 \in \mathcal{P}$ such that $M_1 \neq M_2$, we have $M_1 \cap M_2 = \emptyset$. The purpose of this problem is for you to explore this definition.
 - (a) Give three examples of a partition of $\{A, B, C, D, E\}$. In each of your examples, briefly explain why conditions (i), (ii) and (iii) hold.
 - (b) What partition of $\{A, B, C, D, E\}$ has the most number of members? What partition of $\{A, B, C, D, E\}$ has the least number of members?
 - (c) Give an example of a set $\mathcal{P}$ of subsets of $\{A, B, C, D, E\}$ that satisfies conditions (i) and (ii) but not (iii).
 - (d) Give an example of a collection $\mathcal{P}$ of subsets of $\{A, B, C, D, E\}$ that satisfies conditions (i) and (iii) but not (ii).
 - (e) Give an example of a collection $\mathcal{P}$ of subsets of $\{A, B, C, D, E\}$ that satisfies conditions (ii) and (iii) but not (i).
 - (f) Is it possible to have a partition of a set S if S has exactly one element. Briefly explain.
 - (g) Is it possible to have a partition of a set S if S is the empty set. Briefly explain.
7. Let X be a set and let $\mathcal{H}$ be a set of subsets of X . For elements s, t of X we define the notation $s \approx_{\mathcal{H}} t$, to mean that there is a member of $\mathcal{H}$ that has both s and t as a member.
 - (a) Prove: For any set S and for any partition $\mathcal{P}$ of S and for any $s, t, u \in S$, if $s \approx_{\mathcal{P}} t$ and $t \approx_{\mathcal{P}} u$ then $s \approx_{\mathcal{P}} u$.
 - (b) Suppose that in the previous statement, we replace the phrase “for any partition $\mathcal{P}$ of S ” by “for any set $\mathcal{P}$ of subsets of S ”. Prove that the resulting statement is false.

¹Version 2/5/04

- (c) Now go back to your proof of part a. Suppose that you tried to modify your proof for part a, so that it proves the statement in part b. Since the statement in part b is false, this should be impossible, which means that somewhere in your proof there must be a point where it is crucial that $\mathcal{P}$ is a partition and not just a set of subsets of S . Find where this occurs in your proof. (Note: If you can't do this, then there's something wrong with your proof and you should fix it!)
8. We begin with some definitions. We say that A is a neighbor of B if $(A \setminus B) \cup (B \setminus A)$ consists of exactly one element.

A list of sets is a *neighborly list of sets* if each set on the list (other than the set that begins the list) is a neighbor of the set that immediately precedes it on the list. (Here the word *list* means a finite sequence of items. For example, $(3, 14, 6, 3, 29, 14)$ is a list consisting of 6 items (which, in this example, are integers). Unlike sets, the order in which items appear is important, and the same object may appear more than once as an item of the list. Notice that 14 appears twice on the list. The first time 14 appears, it is immediately preceded by 3. The second time 14 appears it is immediately preceded by 29.)

Here is an interesting theorem: For all positive integers n , there is a list consisting of subsets of $\{1, \dots, n\}$ such that (1) every subset of $\{1, \dots, n\}$ appears *precisely once* on the list and (2) the list is neighborly.

Prove the special cases of the theorem with $n = 1$, $n = 2$, $n = 3$ and $n = 4$.

(Note: A very important part of doing this problem is to carefully read and understand the definitions in the first two paragraphs. If you are having trouble understanding definitions, then one way to help yourself is to do examples. For example, to help yourself understand the definition of neighbor, write down any two sets. Use the definition of neighbors to determine whether the first is a neighbor of the second. Do a few more examples until you understand what neighbors means. Make sure you can construct examples of two sets that are neighbors and of two sets that are not neighbors. To help yourself understand the definition of a neighborly list of sets, write down a list of sets and use the definition to test whether the list you wrote down is a neighborly list of sets. Again, do enough examples to be sure you understand this definition. Once you understand the definitions, you can start thinking about the proofs that you are asked to give.)