Intro to Mathematical Reasoning (Math 300) – Homework 2 [1]

1. This problem is only for those of you who *did not* get the correct minimum number of rounds in the hat switching problem. Since your solution was not the minimum, this means that *there was something wrong with your reasoning.* Analyze your own written argument and find *the exact point in your argument* where you made an untrue statement. Write a brief and specific explanation of what is wrong in your argument. (In most case you should be able to find a particular sentence where you went wrong in your reasoning. It is not enough to say "My argument went wrong because I said the answer was 6 rounds." You must identify the key place in your argument where you (unintentionally) lied.

   Be sure to attach your paper with your solution to the original problem with this homework.

2. Let S denote the set $\{2, 10, 11, 18, 19, 27\}$. For each of the following statements, determine whether the statement is true or false. Give a careful explanation for each answer. (Note: "careful" does not mean "long".)

   Recall that when we say that an integer a is a multiple of b we mean that there is an integer k such that $bk = a$.

   (a) For all elements x belonging to S there is a y belonging to S such that $x+y$ is a multiple of 5.

   (b) There exists an element x belonging to S such that for all elements y belonging to S, $x + y$ is a multiple of 5.

   (c) For all x belonging to S there is a y belonging to S such that $x + y$ is a multiple of 7.

3. Here are some universal statements. For each one, (i) prove that the statement is incorrect, and (ii)if possible, find a simple restriction of the hypothesis of the statement that makes it true. (This is not possible for all of them.) In choosing your restriction, try to add a condition that is least restrictive.

   (a) For any real numbers a, b, c, if $ab = ac$ then $b = c$.

   (b) For any positive integer a, positive integer b and positive integer c, if bc is a multiple of a then b is a multiple of a or c is a multiple of a.

   (c) For any four positive real numbers r, s, t, u if $r \leq s$ and $t \leq u$ then $rs \leq tu$.

   (d) For all positive integers n, $n^4 \geq 2^{n-1}$.

   (e) For any two positive real numbers x, y, $\frac{1}{x} + \frac{1}{y} \leq \frac{4}{x+y}$.

   (f) For any two real numbers y_1, y_2 belonging to the interval $[-\pi, \pi]$, if $\cos(y_1) = \cos(y_2)$ then $y_1 = y_2$.

4. In section 8.1 of the book on page 180 are listed five "bullets" which state some properties of the real numbers. Bullets 1,2 and 5 each give a single universal statement, bullet 3 has two existential statements, and bullet 4 has two universal statements, for a total of 7 statements.

[1] Version 1/19/04

(a) Suppose you replace the words “real number” by “integer” in each of the statements. Which of the seven statements is now false? Explain.

(b) Suppose you replace the words “real number” by “positive integer” in each of the statements. Which of the seven statements is now false? Explain.

5. Consider the statement: “For all pairs x, y of integers, if there is an integer d that is greater than 1 such that x is a multiple of d and y is a multiple of d, then there is an integer m that is less than xy such that m is a multiple of x and m is a multiple of y.

   (a) Draw the parse tree for the above sentence.

   (b) Write an English sentence that is equivalent to the negation of the above sentence and is obtained by pushing negations through.

6. Here is a predicate with free variables x, which stands for a real number and S, which stands for a set of real numbers. “x is not a member of S and for all real numbers $\varepsilon > 0$, there exists a member y of S such that $|x - y| \leq \varepsilon$”.