Intro to Mathematical Reasoning (Math 300) – Assignment 1 [1]

1. Consider a collection of small square tiles. It is known that each tile has a letter on one side and a positive integer on the other side, but no other information is known about the tiles. The tiles are laid out, some with letter side up and some with number side up. Someone tells you, "Every tile that has a consonant on one side has an even number on the other side". Which tiles need to be examined in order to check whether this is true? Explain your answer. (Hint: you should not try to do this problem without making up some examples for yourself and thinking them through.)

2. At a party, 16 people play the following game. The players are numbered from 1 to 16 and each player is given a tag to wear having his or her number. There are also 16 hats numbered 1 to 16. Player 1 is given hat number 16, and every other player is given the hat whose number is one less than the player's number.

   The players now play a game in which they exchange hats as follows. The game is played in rounds. In each round, the players pair up in any way they choose (to form 8 pairs of players). Once the players pair up, each pair of players may either exchange hats or keep their hats. The goal of the game is to finish with each player having the hat whose number matches his or her tag.

   What is the smallest number of rounds needed to accomplish the goal? Give a careful argument justifying your answer.

[1] Version: 1/14/04