Intro to Mathematical Reasoning (Math 300)–Section H2–Fall 2016
Assignment 1 (Preliminary assignment) [1]

The purpose of this assignment is to help me to assess where students are at the beginning of the course. You are expected to make a serious effort to solve the problems and to write up your solutions, but *your scores on these problems will not count towards your final grade.* It is important that you work on these on your own (and not consult other people, books or the internet.)

Problems 1,2 and 3 are "brain teasers" – recreational math puzzles that should be fun to think about. On these problems, you should (1) try to solve them and (2) give a careful explanation of your solution. Please indicate if you were already familiar with the problem.

In problems 4,5 and 6, I ask you to try writing proofs. The problems being asked are problems that you will learn how to solve during the semester, and I want to see if any of you can do some of them already. Make a reasonable effort but *if you can't do them don't worry about it*; I expect that most students will be unable to do most of them,.

1. There is a collection of 50 small square tiles. Each tile has a letter on one side and a positive integer on the other side. No other information is known about the tiles. The tiles are laid out, some with letter side up and some with number side up. Someone asks you "Is it true that every tile that has a consonant on one side has an even number on the other side?" You are supposed to answer this question. If you answer it correctly you will win $50, but if you answer it incorrectly you will lose $200. You may pick up any tiles you want and examine them, but you must pay $1 for each tile you pick up. Which tiles should you pick up and examine?

   Explain your answer.

2. Four people, Bob, Carol, Ted and Alice, are walking in the mountains at night with one flashlight. They arrive at a chasm that is crossed by a flimsy rope bridge. They decide that at most two people can be on the bridge at the same time. , and whoever crosses must have the flashlight with them. None of them can throw the flashlight across the bridge. Therefore they will cross the bridge in five steps as follows:

   1. Two cross the bridge together with the flashlight (so there are two on each side)
   2. One walks back with the flashlight (so there are three at the beginning and one at the end)
   3. Two cross the bridge together with the flashlight (so there is one at the beginning and three at the end)
   4. One walks back with the flashlight (so there are two on each side)
   5. Two cross the bridge together with the flashlight (finished!)

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(Problem 2, continued.) Each person walking alone would need the following amount of time to cross the bridge:

Bob	1 minute
Carol	2 minutes
Alice	5 minutes
Ted	10 minutes

Whenever two people travel together, they go at the speed of the slower person.

Determine the fastest way to get all four people across the bridge. Give a careful argument that your plan is best possible.

3. At a party, 16 people play the following game. The players are numbered from 1 to 16 and each player is given a tag to wear having his or her number. There are also 16 hats numbered 1 to 16. Player 1 is given hat number 16, and every other player is given the hat whose number is one less than the player's number. Everyone can see the numbers on all hats and all tags.

   The players play a game in which they exchange hats as follows. The game is played in rounds. In each round, the players pair up in any way they choose (to form 8 pairs of players). Each pair of players may either exchange hats or keep their hats. The goal of the game is to finish with each player having the hat whose number matches his or her tag.

   What is the smallest number of rounds needed to accomplish the goal? Give a careful argument justifying your answer.

4. Recall that if f and g are functions, then $f = g$ means that f and g have the same domain, and for every member a of the domain $f(a) = g(a)$. In this problem A, B, C, D denote four arbitrary sets. Recall that if $f : A \longrightarrow B$ is a function then we say f is *one-to-one* provided that for all $x, y \in A$, if $x \neq y$ then $f(x) \neq f(y)$. Also if $g : A \longrightarrow B$ and $h : B \longrightarrow C$ then $h \circ g$ is the function from A to C defined for $a \in A$ by $h \circ g(a) = (h(g(a))$.

   Now, suppose $g : B \longrightarrow C$ is a 1-1 function. For each of the following two statements, prove the statement or give a counterexample.

   (a) For any two functions $h_1 : A \longrightarrow B$ and $h_2 : A \longrightarrow B$ if $g \circ h_1 = g \circ h_2$ then $h_1 = h_2$.

   (b) For any two functions $f_1 : C \longrightarrow D$ and $f_2 : C \longrightarrow D$ if $f_1 \circ g = f_2 \circ g$ then $f_1 = f_2$.

5. Prove that for every positive integer n it is possible to find a polynomial $p(x)$ with coefficients belonging to the set $\{-1, 0, 1\}$ such that $n = p(3)$.

6. A set S of real numbers is said to be *α-narrow* (where α is a positive number) if the difference between any two members of S is at most α. Prove that if S is any infinite set of real numbers that is α-narrow, and β is any positive number, then S has an infinite subset T that is β-narrow.