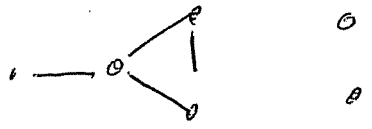
Math 428 - Solutions to (some) review problems for Exam #1 - October 9, 2009

These are solutions to problems that require diagrams (and hence require a scanner to turn into pdf format). Solutions to other problems will be posted separately.

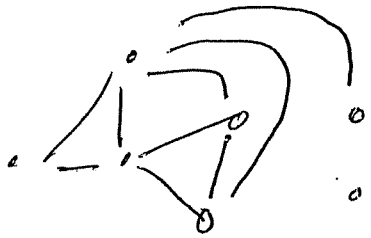
#2(a) $(6,5,3,2,2,1,1,1)$ has an odd number of odd entries, so it is not graphical.

(b) $(7,6,5,4,4,4,3)$ is not graphical for the same reason

(c) $(7,6,5,4,4,3,2,1)$ is graphical if and only if $(5,4,3,3,2,1,0)$ is graphical. Thus is graphical if and only if $(3,2,2,1,0,0)$ is graphical. Thus is graphical, for example

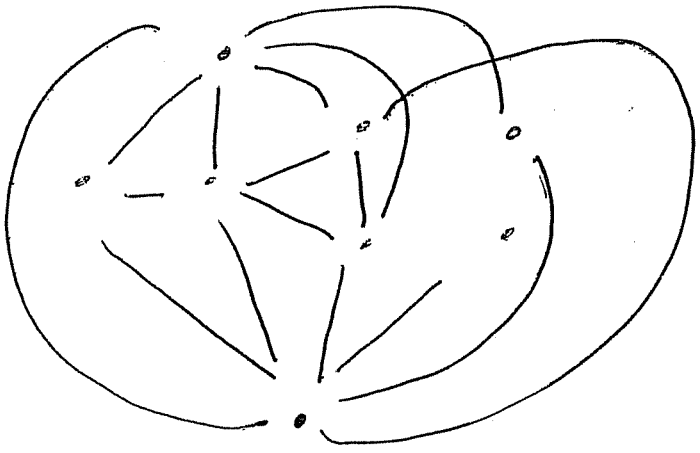


Then



has degree sequence $(5,4,3,3,2,1,0)$

and

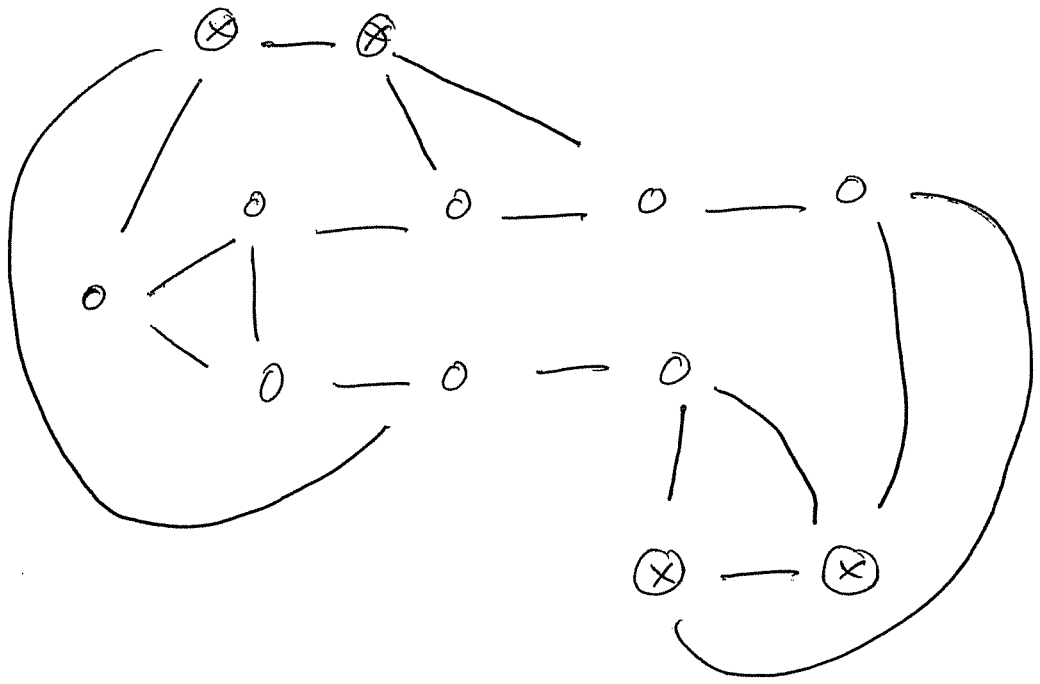


has degree sequence $(7,6,5,4,4,3,2,1)$

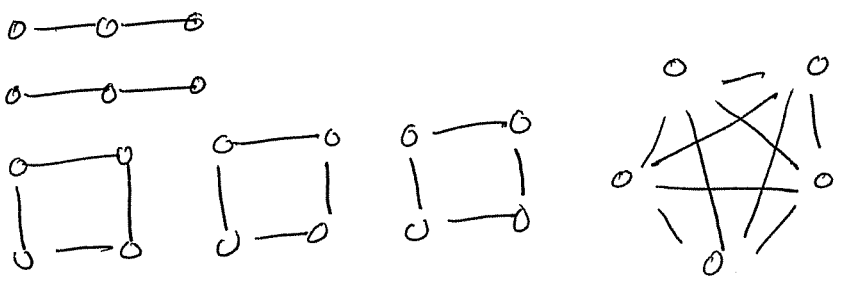
(d) $(7,6,6,5,4,3,2,1)$ is graphical if and only if $(5,5,4,3,2,1,0)$ is graphical. Thus is graphical if and only if $(4,3,2,1,0,0)$ is graphical. But $(4,3,2,1,0,0)$ is graphical if and only if $(2,1,0,-1,0)$ is graphical. Since this is not graphical, neither is $(7,6,6,5,4,3,2,1)$.

#4 Let G be the given graph. It has two vertices of degree 1 and 4 vertices of degree 2. Thus a 3-regular graph

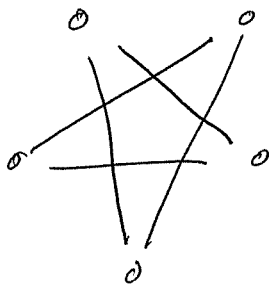
containing G as an induced subgraph must contain at least 2 new vertices (for every vertex of order 1 must be adjacent to two new vertices) and at least 8 new edges (1 for each vertex of order 2 and 2 for each vertex of order 1). But if there are 8 new edges there must be at least $\frac{8}{3}$ new vertices. Thus there must be at least 3 new vertices, so a 3-regular graph containing G as an induced subgraph must have at least 11 vertices. Since the number of vertices in a 3-regular graph must be even (since 3 is odd) there must be at least 12 vertices. Here is such a graph of order 12.



#8 (a)

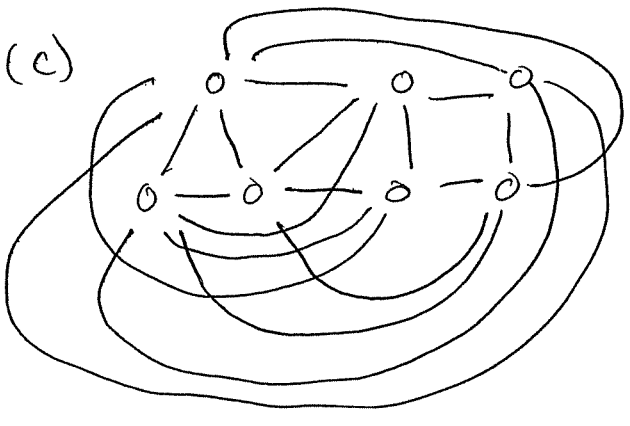


(b)

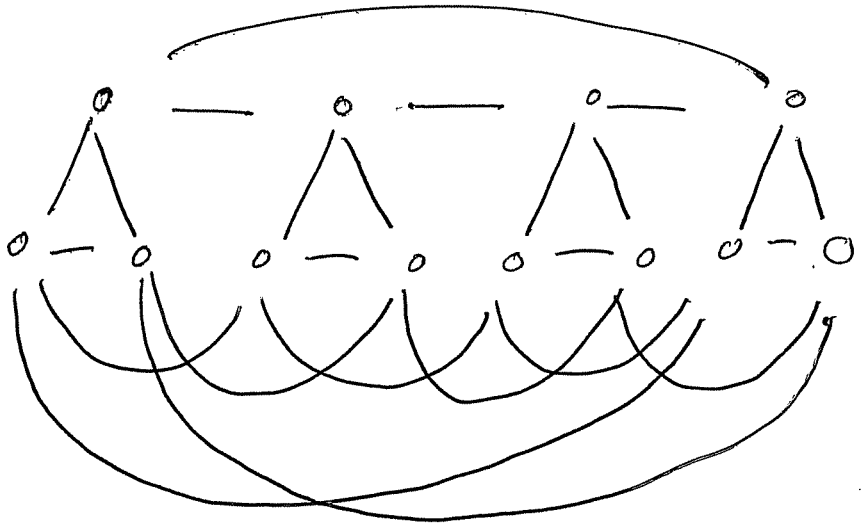


(which is just C_5 again)

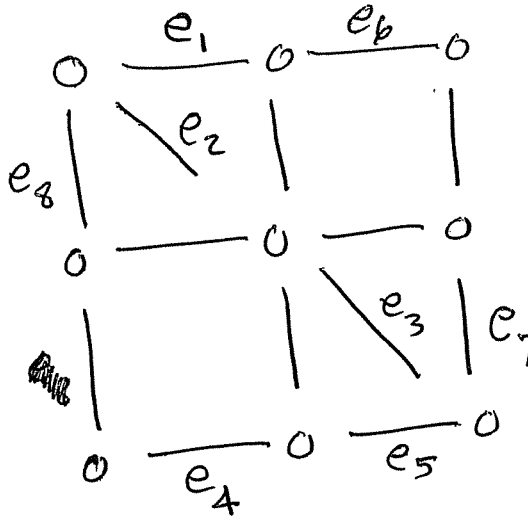
(c)



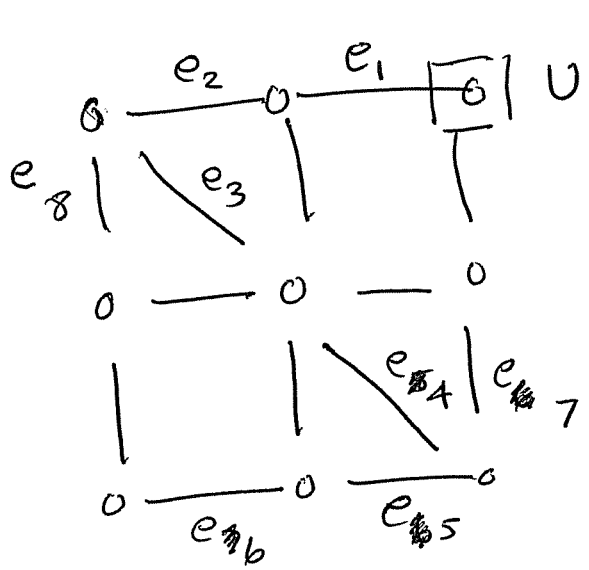
(d)



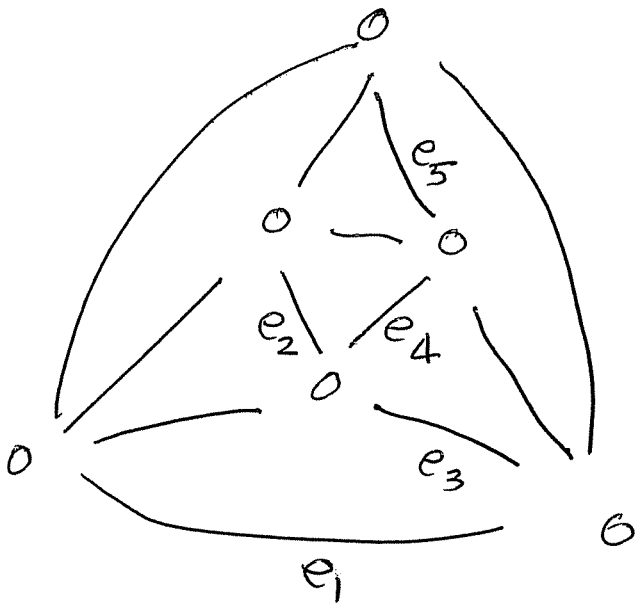
10 (a)



Kruskal

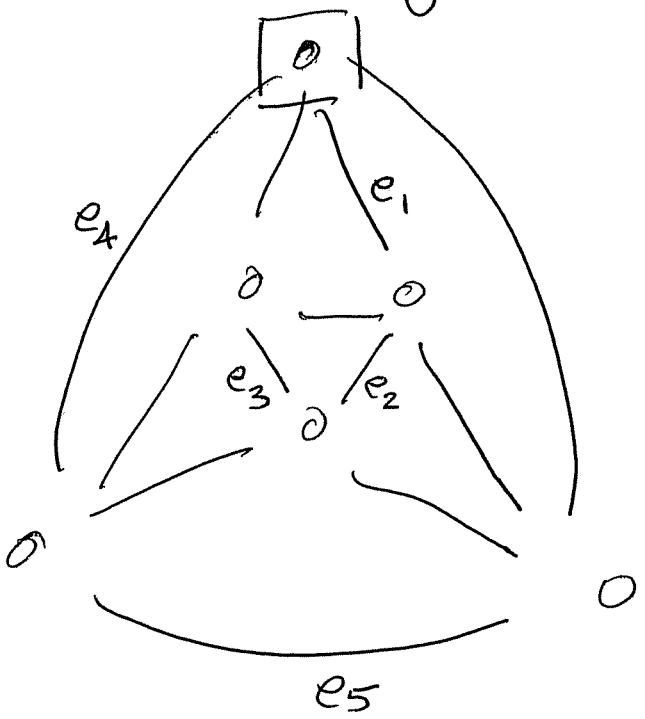


10 (b)



Kruskal

U ← this is arbitrary



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