Advances in
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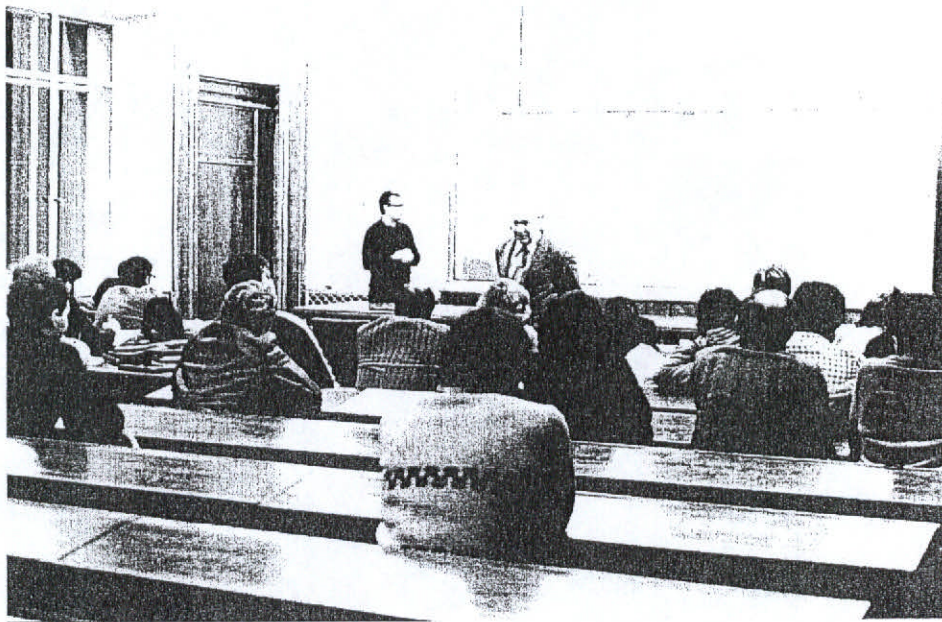
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I. M. Gelfand Seminar

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Photographs courtesy of C. MacPherson

Foreword

September 2, 1993 is Israel Moiseevich Gelfand's 80th birthday. This date practically coincides with the 50th anniversary of the Gelfand Seminar in Functional Analysis at Moscow University. The present volume consists of papers written by "young" participants of this seminar. One of the reasons for introducing an age limit was to keep the volume's size within reasonable bounds, another was Gelfand's constant orientation to the younger participants of his seminar. This collection is intended to be a surprise to the man whose birthday we are celebrating, and I hope that he will learn of the book's existence only after its publication. However, we have tried to imitate Gelfand's own preferences as much as possible. All of the invited authors were participants and welcome speakers at the seminar; if one imagines its Golden Jubilee Session, it may be safely conjectured that these mathematicians would have been invited to participate. I hope that Gelfand will approve our choice and will enjoy seeing articles written by these remarkable mathematicians. The invited authors were free to choose their topics and, if they so desired, their coauthors.

The problem of defining the notion of "young" scientist, in particular, of "young mathematician", is one of the most difficult unsolved problems, and is also of the utmost importance for the applications. In the given case its solution was formalized in the following way. For the upper bound of the age of an invited author to this collection, we chose the age of Serezha, Gelfand's older son, who incidentally played a key role in the appearance of this book. As paradoxal as this may sound, there are serious grounds for this choice. When Serezha first appeared at the seminar in 1961, significant changes in it took place. The orientation to the younger participants that had always been important, but concerned only Ph.D.-track students (occasionally younger graduate students) until then, was now drastically amended to include freshmen and sophomores, and later even some high school students. Among the students of this "first draft" one should note Dima Kazhdan and, somewhat later, Ossya Bernstein. Both became important participants in the seminar for many years. Progressively the other authors of this volume also appeared there. By 1962 the seminar left the relatively small auditorium 13-11 to move to the roomier 14-08. By then no less than half the participants were undergraduates. At that time Gelfand liked to repeat that the seminar was open to all students of the lower courses and to the most talented professors.

I think that it is only natural to include in this foreword some information about the seminar itself. I understand that it was well known in the West. Usually mathematical voyagers from Western Europe or the US felt compelled to visit it (to do that, they had to overcome the vigilance of the Moscow University guards, by no means an easy task even for Muscovites). It is difficult to explain to the Western reader what the seminar meant to "Soviet mathematical life". Surprisingly, that life, in many respects, was not at all so bad, despite the almost unwavering antisemitism and the constantly increasing control by the mathematical rabble with communist party background over the key positions in mathematics. Mathematics was an Oasis of sorts, very attractive to young people with strong interest in science and without career aspirations in the communist hierarchy. If one were lucky, you were able to live an intense intellectual life and write articles free of any references to the classics of Marxism-Leninism. Fortunately for mathematics, Stalin did not find time for the subject (unlike economics, biology, and linguistics).

The Gelfand seminar was always an important event in the very vivid mathematical life in Moscow, and, doubtless, one of its leading centers. A considerable number of the best Moscow mathematicians participated in it at one time or another. Mathematicians from other cities used all possible pretexts to visit it. I recall how a group of Leningrad students agreed to take turns to come to Moscow on Mondays (the day of the seminar, to which other events were linked), and then would retell their friends what they heard there. There were several excellent and very popular seminars in Moscow, but nevertheless the Gelfand seminar was always an event.

I would like to point out that, on the other hand, the seminar was very important in Gelfand's own personal mathematical life. Many of us witnessed how strongly his activities were focused on the seminar. When, in the early fifties, at the peak of antisemitism, Gelfand was chased out of Moscow University, he applied all his efforts to save the seminar. The absence of Gelfand at the seminar, even because of illness, was always something out of the ordinary.

One cannot avoid mentioning that the general attitute to the seminar was far from unanimous. Criticism mainly concerned its style, which was rather unusual for a scientific seminar. It was a kind of a theater with a unique stage director, simultaneously playing the leading role in the performance and organizing the supporting cast, most of whom had the highest qualifications. I use this metaphor with the utmost seriousness, without any intension to mean that the seminar was some sort of a spectacle. Gelfand had chosen the hardest and most dangerous genre: to demonstrate in public how he understood mathematics. It was an open lesson in the grasping of mathematics by one of the most amazing mathematicians of our time. This role could

only be played under the most favorable conditions: the genre dictates the rules of the game, which are not always very convenient for the listeners. This means, for example, that the leader follows only his own intuition in the final choice of the topics of the talks, interrupts them with comments and questions (a priviledge not granted to other participants), organizes their "understanding", mainly by the younger participants (parts of the reports are repeated, participants are summoned to the blackboard). All this is done with extraordinary generosity, a true passion for mathematics.

Let me recall some of the stage director's stratagems. An important feature were improvisations of various kinds. The course of the seminar could change dramatically at any moment. Another important *mise en scène* involved the "trial listener" game, in which one of the participants (this could be a student as well as a professor) was instructed to keep informing the seminar of his understanding of the talk, and whenever the information was negative, that part of the report would be repeated. A well-qualified trial listener could usually feel when the head of the seminar wanted an occasion for such a repetition. Also, Gelfand himself had the faculty of being "unable to understand" in situations when everyone around was sure that everything is clear. What extraordinary vistas were opened to the listeners, and sometimes even to the mathematician giving the talk, by this ability not to understand. Gelfand liked that old story of the professor complaining about his students: "Fantastically stupid students—five times I repeat proof, already I understand it myself, and still they don't get it."

It has remained beyond my understanding how Gelfand could manage all that physically for so many hours. Formally the seminar was supposed to begin at 6 P.M., but usually started with about an hour's delay. I am convinced that the free conversations before the actual beginning of the seminar were part of the scenario. The seminar would continue without any break until 10 or 10:30 (I have heard that before my time it was even later). The end of the seminar was in constant conflict with the rules and regulations of Moscow State University. Usually at 10 P.M. the cleaning woman would make her appearance, trying to close the proceedings to do her job. After the seminar, people wishing to talk to Gelfand would hang around. The elevator would be turned off, and one would have to find the right staircase, so as not to find oneself stuck in front of a locked door on the ground floor, which meant walking back up to the 14th (where else but in Russia is the locking of doors so popular!). The next riddle was to find the only open exit from the building. Then the last problem (of different levels of difficulty for different participants)—how to get home on public transportation, at that time in the process of closing up. Seeing Gelfand home, the last mathematical conversations would conclude the seminar's ritual. Moscow at night was still safe and life seemed so unbelievably beautiful!

The Gelfand seminar exerted a great influence on many mathematicians, not only on his direct pupils. The seminar for all of us was a unique opportunity to witness Gelfand's informal understanding of mathematics, to learn the usually hidden mysteries of the craft. We are all glad that Israel Moiseevich approaches his 80th anniversary in fine health and spirits, has preserved his workaholic ways, and, as always, is bristling with new plans and ideas, on the seventh decade of his service to Mathematics.

                                                            Simon Gindikin