This is the 2nd midterm exam for Math 135, Sections 16-18 in spring 2010.

This exam covered Sections 3.6-4.5. It did __not__ cover Section 4.6.

The 2nd midterm exam for Math 135, Section C7 in Summer 2010 __will__ cover Section 4.6

Math 135 - Sections 16 - 18

Exam #2 Form A   **April 8, 2010**

Instructions: (1) There are eight problems worth a total of 100 points. The number of points assigned to each problem is shown in parentheses after the problem number.

(2) Show your work. No credit will be given for unsupported answers to problems requiring comptation. You may receive credit for partially correct work even if your final answer is incorrect.

(3) The last page of this booklet is a formula sheet

(4) You may use a calculator, but not a laptop computer or any device with a type-writer keyboard.

#1 (9 points) Find $f'(x)$ if $f(x) = (e^x)^{3x}$. Show your work.

Recall $\qquad f'(x) = f(x) \dfrac{d}{dx}\left(\ln f(x)\right)$

$$= (e^x)^{3x} \frac{d}{dx}\left(3x \ln e^x\right)$$

$$= (e^x)^{3x} \frac{d}{dx}\left(3x(x)\right)$$

$$= (e^x)^{3x} \frac{d}{dx}(3x^2)$$

$$= 6x(e^x)^{3x}$$

OR: $\quad (e^x)^{3x} = e^{3x^2}$ so

$$\frac{d}{dx}f(x) = (e^{3x^2})(6x).$$

Please do not write in this space

#1:   #2:   #3:   #4:   #5

#6:   #7   #8:   TOTAL:

1

#2 (12 points) Find each of the following limits. Show your work.

(a) $lim_{x \to 2} \frac{\sin^2(2-x)}{x^3 - 3x^2 + 4}$

$$= \lim_{x \to 2} \frac{-2(\sin(2-x)\cos(2-x))(-1)}{3x^2 - 6x}$$

$$= \lim_{x \to 2} \frac{2(\cos^2(2-x) - \sin^2(2-x))(-1)}{6x - 6}$$

$$= -\frac{2}{6} = -\frac{1}{3}$$

(b) $lim_{x \to 0+} (\frac{x+2}{2x} - \frac{1}{\sin(x)})$

$$= \lim_{x \to 0^+} \frac{(x+2)\sin x - 2x}{2x \sin x}$$

$$= \lim_{x \to 0^+} \frac{\sin x + (x+2)\cos x - 2}{2\sin x + 2x \cos x}$$

$$= \lim_{x \to 0^+} \frac{\cos x + \cos x - (x+2)\sin x}{2\cos x + 2\cos x - 2x\sin x} = \frac{2}{4} = \frac{1}{2}$$

(c) $lim_{x \to \infty}(1 + \frac{5}{x})^x.$

Let $y = \lim_{x \to \infty} (1 + \frac{5}{x})^x$

Then $\ln y = \lim_{x \to \infty} \ln((1+\frac{5}{x})^x) = \lim_{x \to \infty} x \ln(1+\frac{5}{x})$

$$= \lim_{x \to \infty} \frac{\ln(1+\frac{5}{x})}{\frac{1}{x}} = \lim_{x \to \infty} \frac{(\frac{1}{1+\frac{5}{x}})(-\frac{5}{x^2})}{-\frac{1}{x^2}}$$

$$= 5.$$

Thus $\lim_{x \to \infty} (1+\frac{5}{x})^x = e^5$

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#3 (10 points) Use differentials to approximate $(7.97)^{\frac{1}{3}}$. Show your work.

Let $f(x) = x^{1/3}$, $x_0 = 8$, $\Delta x = -.03$

Then $f(x) \approx f(x_0) + f'(x_0) \Delta x$

Here $f'(x) = \frac{1}{3} x^{-2/3}$; $f(8) = 2$, $f'(8) = \frac{1}{3}(8)^{-2/3} = \frac{1}{12}$

Hence $(7.97)^{1/3} = f(7.97) \approx 2 + \frac{1}{12}(-.03)$

$$= 2 + \frac{1}{4}(-.01)$$

$$= 2 - .0025$$

$$= 1.9975$$

#4 (16 points) For each of the following functions find the absolute maximum and absolute minimum values the function on the given interval and give the values of x for which these maximum and minimum values are attained. Show your work.

(a) $f(x) = x^4 - 8x^2 + 7$ on the interval $[-1, 3]$

$$f'(x) = 4x^3 - 16x = 4x(x^2 - 4) = 4x(x+2)(x-2)$$

Thus the critical numbers are $x = -2, 0, 2$. Only 0 and 2 are in the interval $[-1, 3]$.

An absolute maximum or minimum can occur only at an endpoint or a critical number. Hence only at one of $-1, 0, 2, 3$. The values of $f(x)$ at these points are given in the following table:

x	$f(x)$
-1	0
0	7
2	-9
3	16

$\leftarrow$ absolute minimum of -9 at $x = 2$
$\leftarrow$ absolute maximum of 16 at $x = 3$

(b) $f(x) = x + \cos(x)$ on the interval $[0, 2\pi]$

$f'(x) = 1 - \sin x$. This is 0 when $\sin x = 1$. which occurs at $x = \frac{\pi}{2}$. Thus an absolute maximum or minimum can occur only at one of $0, \frac{\pi}{2}, 2\pi$. The values of $f(x)$ at these points are given in the following table:

x	$f(x)$
0	1
$\pi/2$	$\pi/2$
2π	$2\pi + 1$

$\leftarrow$ absolute minimum of 1 at $x = 0$
$\leftarrow$ absolute maximum of $2\pi + 1$ at $x = 2\pi$

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#5 (16 points) A ladder 15 feet long is leaning agains a vertical wall. The bottom of the ladder is sliding away from the wall at the rate of 1 foot per minute. As this happens, the top slides down the vertical wall. How fast is the top sliding down when the bottom of the ladder is 9 feet from the wall?

$\frac{dx}{dt} = 1$

When $x = 9$,

$9^2 + y^2 = 15^2$
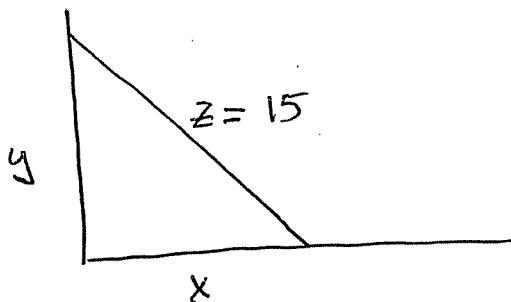
$y^2 = 225 - 81$

$= 144 = 12^2$

so $y = 12$

$x^2 + y^2 = z^2 = 15^2$

so $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

when $x = 9$

$2(9)(1) + 2(12)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = -\frac{9}{12} = -\frac{3}{4}$

The top of the ladder is sliding down at $\frac{3}{4}$ ft/minute.

#6 (16 points) Find all critical numbers for each of the following functions and determine whether each critical number gives a relative maximum, a relative minimum or neither. Show your work and explain how you decide if a critical number gives a relative, a relative minimum or neither. ALSO, determine where the function is concave up and where it is concave down and find the x-values of all points of inflection.

(a) $f(x) = 2x^3 - 3x^2 - 12x + 7$

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x-2)(x+1)$

Thus the critical numbers are -1 and 2

$f'(x) > 0$		$f'(x) < 0$		$f'(x) > 0$
Increasing on $(-\infty, -1]$	-1	decreasing on $[-1, 2]$	2	Increasing on $[2, \infty)$

relative maximum at $x = -1$

relative minimum at $x = 2$

$f''(x) = 12x - 6 = 12(x - \frac{1}{2})$

$f''(x) < 0$		$f''(x) > 0$
concave down on $(-\infty, \frac{1}{2})$	$x = \frac{1}{2}$	concave up on $(\frac{1}{2}, \infty)$
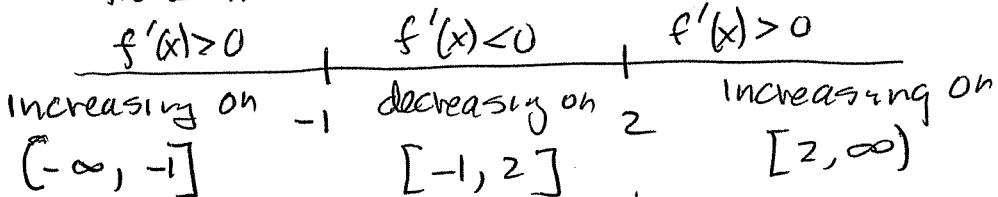
point of inflection at $x = \frac{1}{2}$
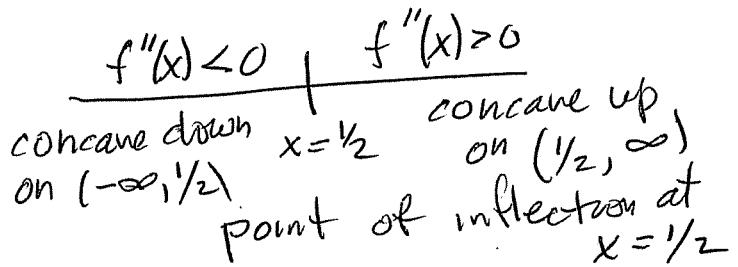
(b) $f(x) = \frac{x^2}{x-3}$

$f'(x) = \frac{2x(x-3) - x^2}{(x-3)^2} = \frac{x(x-6)}{(x-3)^2}$

$f'(x) > 0$		$f'(x) < 0$		$f'(x) < 0$		$f'(x) > 0$
Increasing on $(-\infty, 0]$	0	decreasing on $[0, 3)$	3	decreasing on $(3, 6]$	6	increasing on $[6, \infty)$

relative maximum at $x = 0$

relative minimum at $x = 6$
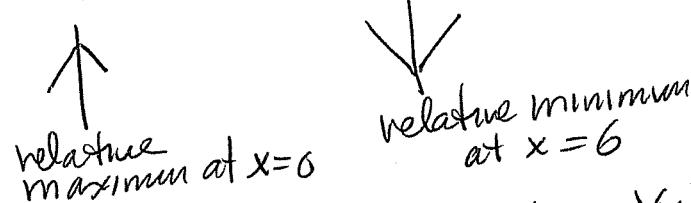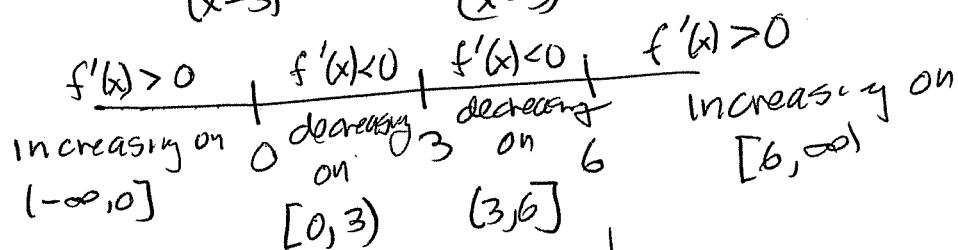
$f''(x) = \frac{(2x-6)(x-3)^2 - 2(x-3)(x^2-6x)}{(x-3)^4} = \frac{(2x-6)(x-3) - 2(x^2-6x)}{(x-3)^3} = \frac{18}{(x-3)^3}$

$f''(x) < 0$		$f''(x) > 0$
Concave down on $(-\infty, 3)$	$x = 3$	concave up on $(3, \infty)$
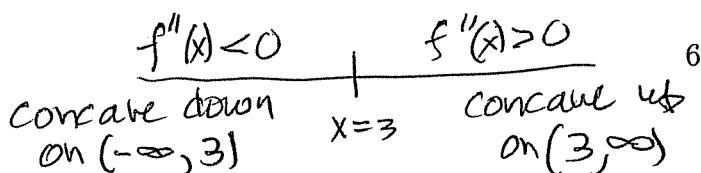
No point of inflection ($\frac{1}{6}$r $f(3)$ is not defined).

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#7 (12 points) Sketch (on the axes below) the graph of a function satisfying ALL of the following conditions. (These conditions do not entirely characterize $f(x)$, so there are many possible answers.)

(i) The domain of $f(x)$ is the set of all real numbers except -1 and 3.

(ii) $lim_{x \to -1-} f(x) = \infty, lim_{x \to -1+} f(x) = -\infty$;

(iii) $lim_{x \to 3-} f(x) = -\infty, lim_{x \to 3+} f(x) = -\infty$;

(iv) $lim_{x \to -\infty} f(x) = lim_{x \to \infty} f(x) = 1$

(v) $f'(x) > 0$ if $x < -1$ or $-1 < x < 1$ or $3 < x < 5$;

(vi) $f'(x) < 0$ if $1 < x < 3$ or $5 < x$;

(vii) $f'(1) = f'(5) = 0$.

$y = 1$

$x = 3$

$x = -1$

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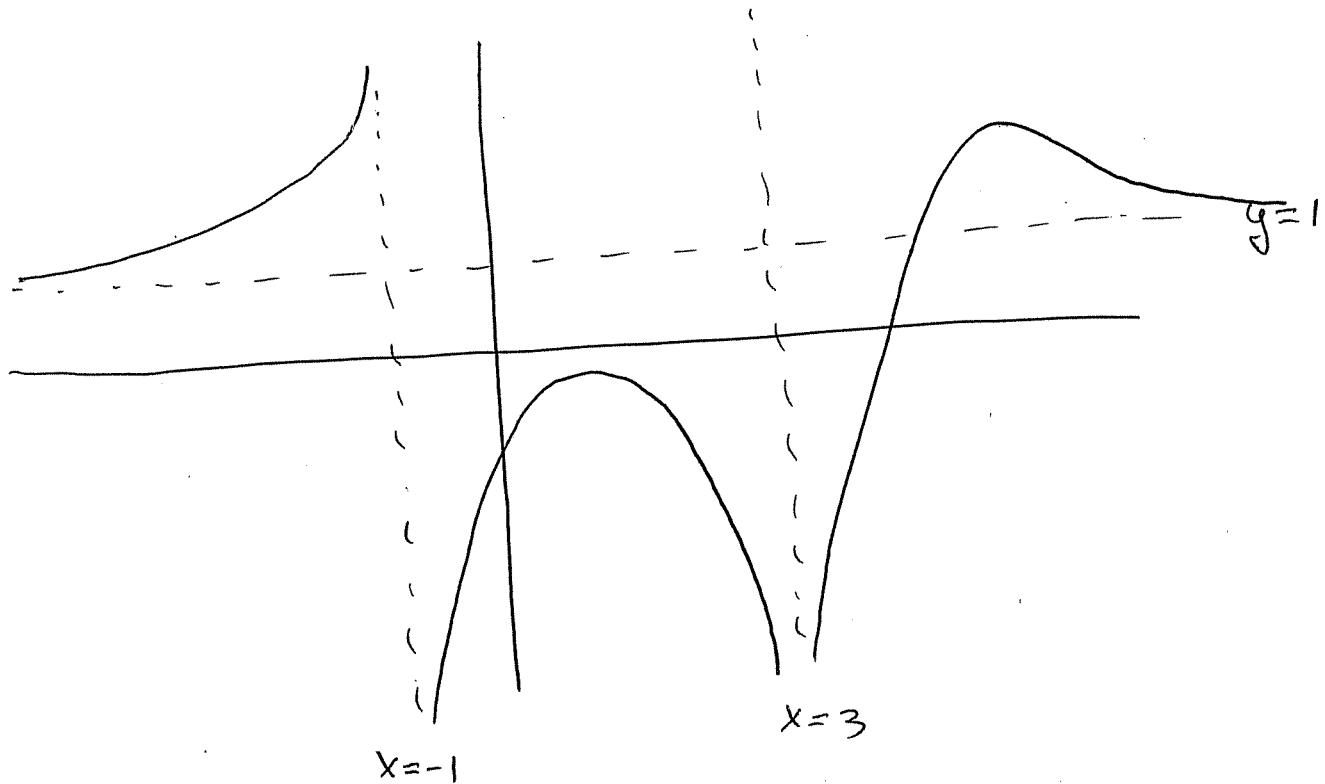
#8 (9 points) Find an equation for the tangent line to the graph of the equation

$$x^3 + y^3 = y + 8$$

at the point $(2, 1)$. Show your work.

$$3x^2 + 3y^2 \frac{dy}{dx} = \frac{dy}{dx}$$

At $(2, 1)$:

$$12 + 3\frac{dy}{dx} = \frac{dy}{dx}$$

$$12 = 2\frac{dy}{dx}$$

$$6 = \frac{dy}{dx}$$