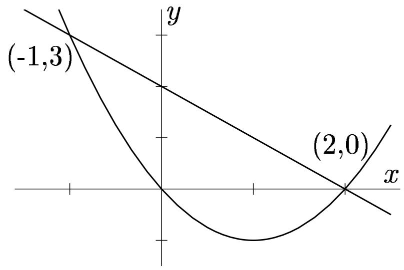
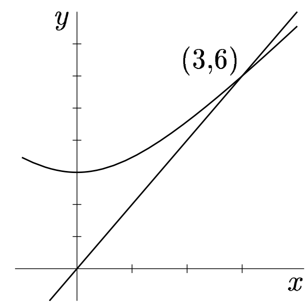
1. The region is bounded above by $y = -x + 2$ and below by $y = x^2 - 2x$. See the picture below.



The line and the parabola cross at those points x where $x^2 - 2x = -x + 2$, or $x^2 - x - 2 = (x-2)(x+1) = 0$. It follows that the points of crossing are at $x = -1$ and $x = 2$. Thus, the area is given by $\int_{-1}^{2} ((-x+2) - (x^2 - 2x))\, dx =$

$$\int_{-1}^{2} (-x^2 + x + 2)\, dx = (-x^3/3 + x^2/2 + 2x)\Big|_{-1}^{2} = 9/2.$$

2. The region is bounded above by $y = \sqrt{9+3x^2}$ and below by $y = 2x$. See the picture at the right. The graphs cross where $2x = \sqrt{9+3x^2}$, or $4x^2 = 9 + 3x^2$, or $x^2 = 9$, or $x = 3$.



2(a) The volume for rotation around the x-axis is:

$$\int_0^3 \pi \left(\sqrt{9+3x^2}\right)^2 dx - \int_0^3 \pi (2x)^2\, dx =$$

$$\int_0^3 \pi(9+3x^2)\, dx - \int_0^3 \pi(4x^2)\, dx = \int_0^3 \pi(9 - x^2)\, dx = 18\pi.$$

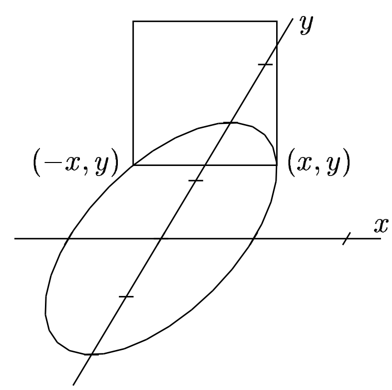
2(b) The volume for rotation around the y-axis is:

$$\int_0^3 (2\pi x)\left(\sqrt{9+3x^2}\right) dx - \int_0^3 (2\pi x)(2x)\, dx =$$

$$2\pi\left(\frac{1}{9}(9+3x^2)^{3/2} - \frac{2}{3}x^3\right)\Big|_0^3$$

$$2\pi\left(\left((36)^{3/2}/9 - 2{\cdot}3^3/3\right) - \left((9)^{3/2}/9 - 2{\cdot}0^3/3\right)\right) = 6\pi.$$

3. Take a cross section of the solid where the y-coordinate is equal to y. Then, the cross section is a square with base in the xy-plane. One end of the base is at (x, y) and the other at $(-x, y)$. See the picture at the right.



Thus, the base of the square has length $2x$. It follows that the area of the square is $4x^2$. Since $4x^2 + y^2 = 4$, the area is $4x^2 = 4 - y^2$.

Then the volume is: $\int_{-2}^{2} (4 - y^2)\, dy = (4y - y^3/3)\Big|_{-2}^{2} =$ 32/3.

4. The average value of $f(x) = \sin^2(x)$ on the interval $[0, 2\pi]$ is given by: $\frac{1}{2\pi}\int_0^{2\pi} \sin^2(x)\, dx =$

$$\frac{1}{2\pi}\int_0^{2\pi} \frac{(1-\cos(2x))}{2}\, dx = \frac{1}{4\pi}(x - \frac{1}{2}\sin(2x))\Big|_0^{2\pi} = \frac{1}{4\pi}\left(2\pi - \frac{1}{2}(\sin(4\pi) - \sin(0))\right) =$$

$$\frac{1}{4\pi}(2\pi - 0)) = \frac{1}{2}$$

5(a) Letting $u = \sin^{-1}(x)$, then $du = dx/\sqrt{1-x^2}$, and the integral becomes $\int u\,du = u^2/2 + C = (\sin^{-1}(x))^2/2 + C$.

5(b) Use integration by parts with $u = \sin^{-1}(x)$, $dv = x\,dx/\sqrt{1-x^2}$, $du = dx/\sqrt{1-x^2}$, $v = -\sqrt{1-x^2}$. Then, $\displaystyle\int \frac{x\sin^{-1}(x)}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2}\sin^{-1}(x) + \int dx = -\sqrt{1-x^2}\sin^{-1}(x) + x + C.$

5(c) Replace $\sin^2(x)$ by $1-\cos^2(x)$ and use the substitution $u = \cos(x)$, $du = -\sin(x)\,dx$ to obtain:

$$\int (1-\cos^2(x))\cos^4(x)\sin(x)\,dx = \int -(1-u^2)u^4\,du = -\frac{1}{5}u^5 + \frac{1}{7}u^7 + C =$$
$$-\frac{1}{5}\cos^5(x) + \frac{1}{7}\cos^7(x) + C.$$

5(d) Since the equation $x^2-4x+8=0$ has complex roots, we complete the square: $x^2-4x+8 = (x-2)^2+4$. Thus, we use the substitution $u = x-2$, $du = dx$. Then, $\displaystyle\int_0^4 \frac{x}{x^2-4x+8}\,dx =$
$$\int_{-2}^{2}\frac{(u+2)du}{u^2+4} = \int_{-2}^{2}\frac{u\,du}{u^2+4} + 2\int_{-2}^{2}\frac{du}{u^2+4} = \left(\frac{1}{2}\ln(u^2+4) + \tan^{-1}(u/2)\right)\Big|_{-2}^{2} = \pi/2.$$

5(e) Use the substitution $x = 2\tan\theta$, then $dx = 2\sec^2\theta\,d\theta$, and $(4+x^2)^{1/2} = 2\sec\theta$. Thus,
$$\int \frac{dx}{(4+x^2)^{3/2}} = \int\frac{2\sec^2\theta\,d\theta}{8\sec^3\theta} = \frac{1}{4}\int\frac{d\theta}{\sec\theta} = \frac{1}{4}\int\cos\theta\,d\theta = \frac{1}{4}\sin(\theta) + C = \frac{1}{4}\frac{x}{\sqrt{4+x^2}} + C.$$

5(f) Let $u = e^x+1$, $du = e^x\,dx$, $e^x = u-1$. Then, $\displaystyle\int\frac{e^{2x}}{(1+e^x)^{1/3}}\,dx = \int\frac{e^x\cdot e^x}{(1+e^x)^{1/3}}\,dx =$
$$\int\frac{(u-1)du}{u^{1/3}} = \int (u^{2/3}-u^{-1/3})\,du = \frac{3}{5}u^{5/3} - \frac{3}{2}u^{2/3} + C = \frac{3}{5}(1+e^x)^{5/3} - \frac{3}{2}(1+e^x)^{2/3} + C.$$

5(g) Here, $x^4+3x^3+2x^2+x-3 = (x^4-1)+3x^3+2x^2+x-2$. Using partial fractions, $\displaystyle\frac{3x^3+2x^2+x-2}{(x-1)(x+1)(x^2+1)} = \frac{1}{x-1} + \frac{1}{x+1} + \frac{x+2}{x^2+1}$. Thus, $\displaystyle\int\frac{x^4+3x^3+2x^2+x-3}{x^4-1}\,dx =$
$$\int\left(1 + \frac{1}{x-1} + \frac{1}{x+1} + \frac{x}{x^2+1} + \frac{2}{x^2+1}\right)dx =$$
$$x + \ln|x-1| + \ln|x+1| + \frac{1}{2}\ln(x^2+1) + 2\tan^{-1}x + C.$$

6. In the problem, $f(x) = \cos(x^2)$, $f'(x) = -2x\sin(x^2)$, and $f''(x) = -2\sin(x^2) - 4x^2\cos(x^2)$. First we determine an upper bound K for $f''(x)$ on the interval $[0,1]$. On $[0,1]$, $|x^2| \le 1$, $|\sin(x^2)| \le 1$, and $|\cos(x^2)| \le 1$. Thus, $|f''(x)| = |-2\sin(x^2) - 4x^2\cos(x^2)| \le 2|\sin(x^2)| + 4|x^2||\cos(x^2)| \le 2\cdot 1 + 4\cdot 1 = 6$. Thus we may take $K = 6$.

When the midpoint rule is used, an upper bound for the error is $K(b-a)^3/24n^2$. In this case, $K = 6$ and $b-a = 1$, and an upper bound for the error is $1/4n^2$. To insure that the error is at most $1/10^6$, we choose n sufficiently large so that $1/4n^2 \le 1/10^6$, or $10^6 \le 4n^2$, or $10^3 \le 2n$, or $500 \le n$.

7. (a), (c) converge; (b), (d) diverge.

7(a) $\displaystyle\int_1^\infty x^{-3/2}\,dx = -2x^{-1/2}\Big|_1^\infty = 2.$

7(b) $\displaystyle\int_0^1 (1-x)^{-3/2}\,dx = 2(1-x)^{-1/2}\Big|_0^1 = \infty.$

7(c) $\displaystyle\int_2^\infty\frac{dx}{x^2-1} = \int_2^\infty\frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right)dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\Big|_2^\infty = 0 - \frac{1}{2}\ln(1/3) = \frac{1}{2}\ln 3.$

7(d) $\displaystyle\int_2^{\infty} \frac{x\,dx}{x^2-1} = \frac{1}{2}\ln|x^2-1|\Big|_2^{\infty} = \infty.$

8(a) The area is $\displaystyle\int_0^{\infty} e^{-x}\,dx = -e^{-x}\Big|_0^{\infty} = 1.$

8(b) Suppose that $x = a$ divides R into two regions of the same area. Then, $\displaystyle\int_0^{a} e^{-x}\,dx = -e^{-x}\Big|_0^{a} = -e^{-a} + 1 = 1/2,$ or $e^{-a} = 1/2$, or $\ln(e^{-a}) = \ln(1/2)$, or $-a = -\ln 2$, or $a = \ln 2$.