(1) Find all integer solutions \((x, y)\) to the equation \(xy - 3x + 2y = 12\). \((Hint: \text{Rearrange the equation to obtain a factorization.})\)

(2) Let \(n\) be a positive integer that has 6 and 8 as factors. What other factors must \(n\) have?

(3) Let \(n \in \mathbb{N}\) and let \(p\) be prime. Prove that \(p\) cannot divide both \(n\) and \(n + 1\).

(4) Let \(n\) be a natural number with prime decomposition \(n = p_1^{s_1}p_2^{s_2}\ldots p_k^{s_k}\). Prove that if \(n = m^2\) for some natural number \(m\) then \(s_1, s_2, \ldots, s_k\) are all even. \((Hint: \text{You can use the Prime Factorization theorem for natural numbers.})\)

(5) Let \(n \in \mathbb{N}_{>1}\). Prove that if \(n\) divides \((n-1)!\) then \(n\) has a proper divisor \(d > 1\).

(6) Prove that if \(n\) is composite, \(n = k\ell\), with \(1 < k < n\) and \(1 < \ell < n\), then \(k\) and \(\ell\) both divide \((n-1)!\).

(7) Prove that every prime number \(p \in \mathbb{P}_{>3}\) is either of the form of \(4n + 1\) or of the form of \(4n + 3\) for some \(n \in \mathbb{N}\).

(8) Prove that if \(p \in \mathbb{P}_{\geq5}\) then \(p^2 + 2\) is composite.

(9) Let \(a, n \geq 2\) be integers. Prove that if \(a^n - 1\) is prime, then \(a = 2\) and \(n\) is prime. \((Hint: \text{use the identity } x^n - 1 = (x - 1)(x^{n-1} + x^{n-2} + \cdots + x + 1) \text{ for any integer } m \geq 2. \text{ This is another chance to write up the complete solution after the discussion in Workshop 4.})\)