640:300 WORKSHOP 2
ODD AND EVEN INTEGERS

We follow Euclid’s definitions of even numbers (The Elements: Book VII: Def. 6) and odd numbers (The Elements: Book VII: Def. 7):

An even number is that which is divisible into two equal parts.

Let \( E \) denote the set of even integers. We may write
\[
E = \{ x \in \mathbb{Z} \mid (\exists y \in \mathbb{Z})(x = 2y) \}.
\]

Then
\[
E = \{ \cdots - 6, -4, -2, 0, 2, 4, 6 \ldots \}.
\]

An odd number is that which is not divisible into two equal parts. Let \( O \) denote the set of odd integers. Then
\[
O = \mathbb{Z} - E = \{ x \in \mathbb{Z} \mid x \notin E \}
\]

and
\[
O = \{ \cdots - 5, -3, -1, 1, 3, 5 \ldots \}.
\]

Prove the following theorem.

**Theorem** Let \( O' = \{ z \in \mathbb{Z} \mid (\exists y \in \mathbb{Z})(z = 2y + 1) \} \). Then \( O' \subseteq O \).

Recall that to prove \( O' \subseteq O \) we must prove that \( x \in O' \implies x \in O \).

**Hint:** Use proof by contradiction.