Calculus 1000A — Fall 2015
Solutions to Written Assignment 3

Due Date: Nov. 04, 2015 (in class) **Name:** ________________________
Section: 007

- There are two problems in this assignment. Each problem can earn you a maximum of 10 points.
- **Attach extra sheets if necessary.**

Problem 1. A particle is moving on the hyperbola $x^2 - 18y^2 = 9$. At what point(s) on the hyperbola is the x-coordinate of the particle moving three times more rapidly than the y-coordinate, assuming that that neither of these velocities are zero?

Solution. Let $(x(t), y(t))$ denote the postion of the particle at time t. Then, since this point is on the given hyperbola, we have that

$$x(t)^2 - 18y(t)^2 = 9.$$

Differentiating both sides of the above equation w.r.t. t, we get

$$2x(t)\dot{x}(t) - 36y(t)\dot{y}(t) = 0,$$

or,

$$x\frac{dx}{dt} = 18y\frac{dy}{dt}.$$

Now, if we substitute $\left|\frac{dx}{dt}\right| = 3\left|\frac{dy}{dt}\right|$ or $\frac{dx}{dt} = \pm 3\frac{dy}{dt}$ in the above equation, we get that

$$3x\frac{dy}{dt} = \pm 18y\frac{dy}{dt},$$

or

$$x = \pm 6y, \qquad \text{since } \frac{dy}{dt} \neq 0.$$

So, if (x, y) is a point on the hyperbola where the x-coordinate of the particle is moving three times more rapidly than the y-coordinate, the point satisfis the following equations:

$$x = \pm 6y \tag{1}$$

and

$$x^2 - 18y^2 = 9. \tag{2}$$

Subsitituting (1) in (2), we get that

$$36y^2 - 18y^2 = 9$$

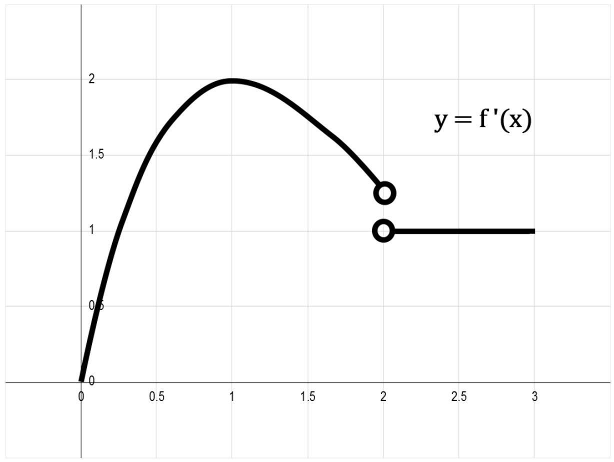
or

$$y = \pm\frac{1}{\sqrt{2}}.$$

So, the points on the hyperbola where the x-coordinate of the particle is moving three times more rapidly than the y-coordinate are

$\boxed{(3\sqrt{2}, 1/\sqrt{2}),\ (-3\sqrt{2}, 1/\sqrt{2}),\ (3\sqrt{2}, -1/\sqrt{2}) \text{ and } (-3\sqrt{2}, -1/\sqrt{2})}$.

Problem 2. Given below is the graph of $f'(x)$, the **derivative** of the function $f(x)$ defined on $[0, 3]$. Answer the following questions and give a one-line justification for each answer.



1. $\lim_{x\to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x\to 2^+} \frac{f(x) - f(2)}{x - 2}$.

   TRUE   [FALSE]   CANNOT BE DETERMINED

   Reason: No value for $f'(2)$ — i.e., derivative does not exist at 2.

2. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$.

   TRUE   FALSE   [CANNOT BE DETERMINED]

   Reason: f can have a discontinuity or can have a continuous corner at 2.

3. $f(x)$ does not have any inflection points in $[0, 3]$.

   TRUE   [FALSE]   CANNOT BE DETERMINED

   Reason: f' is increasing in $(0, 1)$ and decreasing in $(1, 2)$, so the concavity of f changes from concave up to concave down at 1, which is, therefore, an inflection point of f.

4. Suppose $f(0) = 0$. Then, $f(x)$ must be positive in $(0, 2)$.

   [TRUE]   FALSE   CANNOT BE DETERMINED

   Reason: $f' > 0$ in $(0, 2)$, so f has to be increasing. Since $f(0) = 0$, $f(x) > 0$ for x in $(0, 2)$.

5. $f(x)$ is never concave down.

   TRUE   [FALSE]   CANNOT BE DETERMINED

   Reason: f' is decreasing in $(1, 2)$, so $f'' < 0$ there. So, f is concave down there.