MID-TERM EXAM: Nov. 02, 2015

MC

$$\lim_{x\to-\infty} \frac{x^3(4-x^{1/3})}{5x^3-3x^2+1}$$

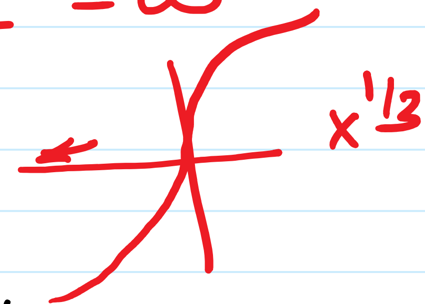
$$= \lim_{x\to-\infty} \frac{\frac{\cancel{x^3}(4-x^{1/3})}{\cancel{x^3}}}{\frac{5x^3-3x^2+1}{x^3}}$$

$$= \lim_{x\to-\infty} \frac{4-x^{1/3}}{5-\frac{3}{x}+\frac{1}{x^3}}$$

$$= \frac{\lim_{x\to-\infty} 4 - \lim_{x\to-\infty} x^{1/3}}{\lim_{x\to-\infty} 5 - \lim_{x\to-\infty} \frac{3}{x} + \lim_{x\to-\infty} \frac{1}{x^3}}$$

$$= \frac{4-(-\infty)}{5-0+0} = \underline{\underline{+\infty}}$$

$$\lim_{x\to-\infty} x^{1/3} = -\infty$$



__Alternatively:__

$$\lim_{x\to-\infty} \frac{(4x^3-x^{4/3})/x^3}{(5x^3-3x^2+1)/x^3}$$

$$= \lim_{x\to-\infty} \frac{4-x^{1/3}}{5-\frac{3}{x}+\frac{1}{x^3}}$$

PURVI GUPTA
SEC 007
7:00 - 9:00pm

MC: Find the domain of

$f(x) = \cos^{-1}(e^x)$.

outside - start here!

domain of $\cos^{-1}$: $[-1, 1]$

So, $\underbrace{-1 \le e^x}_{\text{e^x is always positive}} \le 1$

Isolate x.

$e^x \le 1$

$\ln(e^x) \le \ln(1)$

$\boxed{x \le 0}$ $\quad (-\infty, 0]$

MC: If $f(x) = 2x-3$ and g is a function such that $g''(0) = 4$ and $(fg)''(0) = -4)$ then $g'(0) = ?$

Product g f and g

Solution: $f(x) = 2x-3$

$f'(x) = 2$

$f''(x) = 0$

$f(0) = 2\cdot 0 - 3 = -3$

$f'(0) = 2$

$f''(0) = 0$

$g''(0) = 4.$

$(fg)''(0) \rightarrow$ First expand

$$(fg)'(0) \stackrel{P.R.}{=} f'(0)\cdot g(0) + f(0)g'(0)$$

$$(fg)''(0) = \left(\underline{f'(0)\cdot g(0)} + \underline{f(0)g'(0)}\right)'$$

$$-4 = f''(0)\cdot g(0) + f'(0)\cdot g'(0) + f'(0)g'(0) + f(0)g''(0)$$

$$-4 = 0\cdot g(0) + 2g'(0) + 2g'(0) + (-3)(4)$$

$$-4 = 4g'(0) - 12$$

$$8 = 4g'(0) \rightarrow \boxed{g'(0) = 2}$$

MC

$$\frac{d}{dx} e^{\arctan(\sqrt{2})}$$

$-\frac{\pi}{2}$ $\quad -\sqrt{2}$ $\quad \underline{\underline{0}}$ $\quad \sqrt{2}$ $\quad \tan\sqrt{2}$

$e^{\arctan(\sqrt{2})}$ is a constant!

$$\frac{d}{dx} e^{\arctan(\sqrt{2})} = 0$$

$$\lim_{\star \to 0} \frac{\sin(\star)}{(\star)} = 1 \quad \to \quad \lim_{\star \to 0} \frac{\star}{\sin(\star)} = 1$$

MC $\displaystyle \lim_{x \to 0^+} \frac{\sin(4x)}{\sqrt{x}\sin(2\sqrt{x})}$

Try substitution : $\frac{0}{0}$

$$\lim_{x \to 0^+} \frac{\sin(4x)}{\sqrt{x}\sin(2\sqrt{x})} \quad \frac{4x}{4x}$$

$$= \lim_{x \to 0^+} \frac{\sin 4x}{4x}\left[\frac{4x}{\sqrt{x}\sin(2\sqrt{x})}\right]$$

$$= \lim_{x \to 0^+} \frac{\sin 4x}{4x}\left[\frac{4x}{\sqrt{x}\sin(2\sqrt{x})}\frac{2\sqrt{x}}{2\sqrt{x}}\right]$$

$$= \lim_{x \to 0^+} \underbrace{\frac{\sin 4x}{4x}}_{\to 1} \underbrace{\frac{2\sqrt{x}}{\sin(2\sqrt{x})}}_{\to 1} \frac{4\cancel{x}}{2\cancel{x}}$$

$$= \underline{\underline{2}}$$

$$f(x) = \underbrace{\sin}_{\text{outside}}\Big(\pi \overbrace{\frac{2-e^x}{2+e^x}}^{\text{inside}}\Big)$$

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this is a composition! not a product!

CHAIN RULE :

$$\cos\left(\pi \frac{2-e^x}{2+e^x}\right) \frac{d}{dx} \pi \frac{2-e^x}{2+e^x}$$

$$\cos\left(\pi \frac{2-e^x}{2+e^x}\right) \pi \boxed{\frac{(2+e^x)(-e^x) - (2-e^x)(e^x)}{(2+e^x)^2}}$$

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$$f(x) = \underbrace{\arcsin}_{\text{outside}}\big(\underbrace{e^{7/x}}_{\text{inside}}\big)$$

$$f'(x) = \frac{1}{\sqrt{1-(e^{7/x})^2}} \cdot \frac{d}{dx} e^{7/x}$$

another chain rule!

$$= \frac{1}{\sqrt{1-(e^{7/x})^2}}\, e^{7/x} \cdot \left(\frac{-7}{x^2}\right)$$

$$\left(\frac{7}{x} = 7x^{-1}\right)$$

4·3 Continued. 11-2-2015

Sketch the graph of

$$f(x) = e^{\sqrt[3]{x}}.$$

Solution:: $f'(x) = \frac{1}{3}x^{-\frac{2}{3}} e^{x^{1/3}}$.

$$f''(x) \overset{P.R.}{=} \frac{1}{3}\left[-\frac{2}{3}x^{-5/3} e^{x^{1/3}} + x^{-2/3}\left(\frac{1}{3}x^{-2/3} e^{x^{1/3}}\right)\right]$$

$$= \frac{1}{3}\cdot\frac{1}{3} e^{x^{1/3}} \left(-2x^{-5/3} + x^{-4/3}\right)$$

$$= \frac{1}{9} e^{x^{1/3}} \left(\frac{-2}{x^{5/3}} + \frac{1}{x^{4/3}} \right)$$

$$= \frac{1}{9} e^{x^{1/3}} \cdot \frac{1}{x^{4/3}} \left(\frac{-2}{x^{1/3}} + 1 \right)$$

$$\left[\frac{1}{x^{2/3}} = \frac{1}{(x^{1/3})^2} \right]$$

CRITICAL NUMBERS:

$f'(0)$ is not defined.

0 is a critical number.

$f'(x) > 0$ for all $x \neq 0$

f is increasing when $x \neq 0$

$$\lim_{x\to 0} f'(x) = \lim_{x \to 0} \frac{1}{3} e^{x^{1/3}} \cdot x^{-2/3}$$

$= \infty$ vertical tangent line!

CONCAVITY:

$$f''(x) =$$

$$\frac{1}{9}e^{x^{1/3}} \cdot \frac{1}{x^{4/3}}\left(\frac{-2}{x^{1/3}} + 1\right)$$

$f''(0)$ is not defined

$$f''(x) = 0$$

$$\cancel{\frac{1}{9}e^{x^{1/3}}}\ \cancel{\frac{1}{x^{4/3}}}\left(-\frac{2}{x^{1/3}} + 1\right) = 0$$

$$-\frac{2}{x^{1/3}} + 1 = 0$$

$$1 = 2/x^{1/3}$$

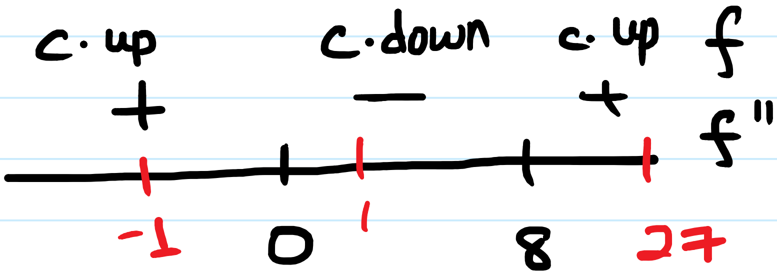
$$x^{1/3} = 2 \text{ OR } \boxed{x = 8}$$

Potential inflection points are @ 0, 8

	c. up	c. down	c. up	f
	$+$	$-$	$+$	f''
-1	0	1	8	27

	$e^{x^{1/3}}$	$\frac{1}{x^{4/3}}$	$\left(1 - \frac{2}{x^{1/3}}\right)$
$x<0$	$+$	$+$	@-1: $1 - \frac{2}{-1} = 1+2 = 3$, $+$
$0<x<8$	$+$	$+$	$-$
$x>8$	$+$	$+$	@ 27: $1 - \frac{2}{3} = \frac{1}{3}$, $+$

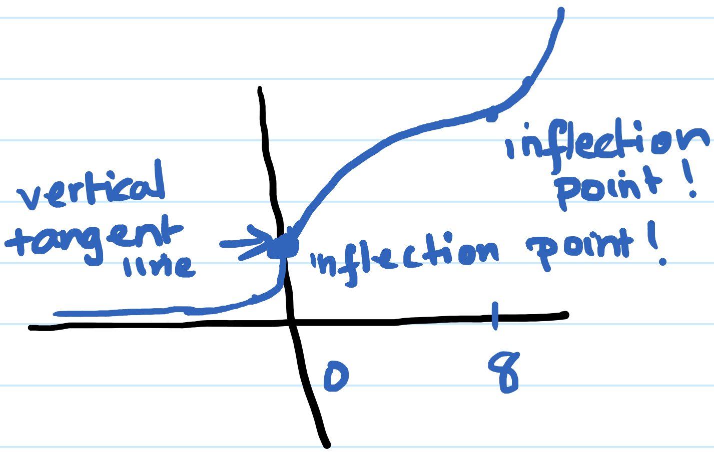
$$\frac{1}{x^{4/3}} = \frac{1}{(x^{1/3})^4}$$



$f(x) = e^{x^{1/3}}$

① always positive!

② $f(0) = e^0 = 1$



$$\lim_{x\to\infty} e^{\sqrt[3]{x}}$$

$t = \sqrt[3]{x}$

$x \to \infty, \quad t \to \infty$

$$\lim_{t\to\infty} e^t = \infty$$

$$\lim_{x\to-\infty} e^{\sqrt[3]{x}}$$

$t = \sqrt[3]{x}$

$x \to -\infty$

$t = \sqrt[3]{x} \to -\infty$

$$\lim_{t\to-\infty} e^t = 0$$

HINT FOR WA 3 Problem 1.

$$\left|\frac{dx}{dt}\right| = \left|3\frac{dy}{dt}\right|$$

Correct condition!

$$\frac{dx}{dt} = 3\frac{dy}{dt} \qquad \frac{dx}{dt} = -3\frac{dy}{dt}$$

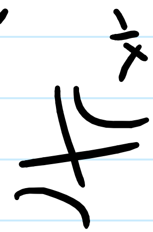
Sketch the graph of

$$f(x) = e^{1/x}.$$

Solution: The domain of $f(x)$ is $x \neq 0$.

@ $x=0$, look for vertical asymptotes!

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{1/x}$$



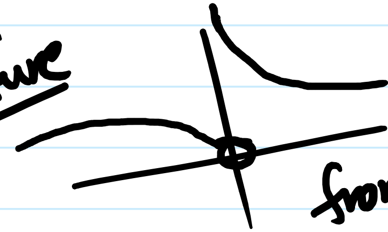
$t = \frac{1}{x}$, $x \to 0^+$

$t \to \infty$

$$\lim_{x\to 0^+} e^{1/x} = \lim_{t\to\infty} e^t = \infty$$

vertical asymptote is $x=0$

rough picture



$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} e^{1/x}$$

$t = \frac{1}{x}$, $x \to 0^-$

$t = \frac{1}{x} \to -\infty$

$$\lim_{t\to-\infty} e^t = 0$$

$$\lim_{x\to 0^-} e^{1/x} = 0$$

$$f'(x) = e^{1/x}\left(\frac{-1}{x^2}\right) = \frac{-e^{1/x}}{x^2}$$

$f'(0)$ is not defined!

0 is the only critical number!

$$f'(x) = e^{1/x}\left(-\frac{1}{x^2}\right) = -\frac{e^{1/x}}{x^2} = -e^{1/x}\cdot x^{-2}$$

$f'(0)$ is not defined!

0 is the only critical number!

$$\begin{array}{ccc} - & & - \\ \hline & 0 & \end{array}$$

	$-e^{1/x}$	$\frac{1}{x^2}$	f'
$x<0$	$-$	$+$	$-$
$x>0$	$-$	$+$	$-$

function is <u>decreasing</u> for all $x \neq 0$

$$f''(x) = -\left[e^{1/x}\left(-\frac{1}{x^2}\right)x^{-2} + e^{1/x}\cdot -2x^{-3}\right]$$

$$= -e^{1/x}\left[-\frac{1}{x^4} - \frac{2}{x^3}\right]$$

$$= e^{1/x}\left[\frac{1+2x}{x^4}\right]$$

$f''(0)$ not defined

$$f''(x) = 0 \to e^{1/x}\left[\frac{1+2x}{x^4}\right] = 0$$

$$\to \cancel{e^{1/x} = 0} \quad \text{OR} \quad \cancel{x^4}\cdot\frac{1+2x}{\cancel{x^4}} = 0\cdot x^4$$

$$\Rightarrow \quad 1+2x = 0$$

$$\Rightarrow \quad x = -1/2$$

Potential inflection pts: $0, -\frac{1}{2}$

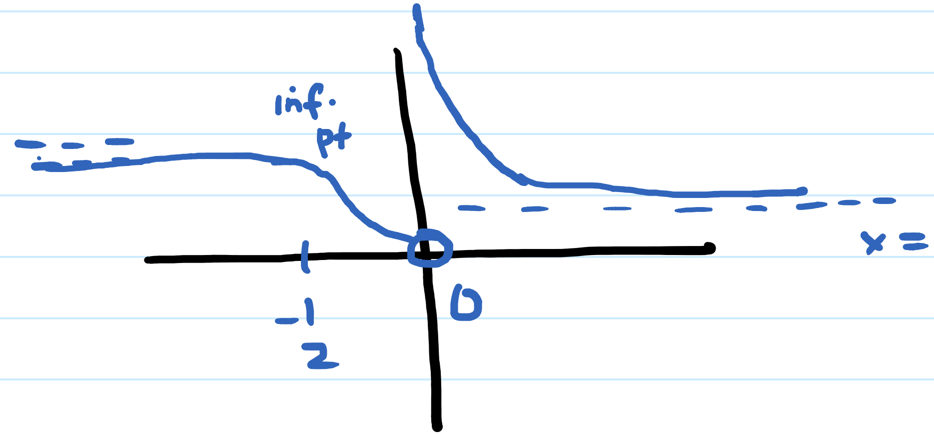
$$e^{1/x}\left[\frac{1+2x}{x^4}\right] = f''(x)$$

c.down. c.up c.up

$-$ $+$ $+$

-1 $\quad -1/2 \quad -\frac{1}{4} \quad 0 \quad 1$

	$e^{1/x}$	$1+2x$	$\frac{1}{x^4}$
$x < -\frac{1}{2}$	$+$	$-$	$+$
$-\frac{1}{2} < x < 0$	$+$	$+$	$+$
$x > 0$	$+$	$+$	$+$



$f(x) = e^{1/x} \qquad > 0$

$$\lim_{x\to\infty} e^{1/x} \qquad t = 1/x$$

$$x \to \infty \quad t \to 0$$

$$\lim_{t\to 0} e^{t} = 1$$

$$\lim_{x\to -\infty} e^{1/x} \qquad t = 1/x$$

$$x \to -\infty,\ t \to 0$$

$$\lim_{t\to 0} e^{t} = 1$$

$\boxed{\text{HW}}$: SECOND-DERIVATIVE TEST FROM 4.3!!