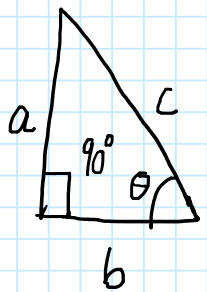
CALCULUS 1000A

SECTION 007
M,W - WSC 55
7:00 - 9:00 PM
INSTRUCTOR: PURVI GUPTA (MC 120)
http://www-home.math.uwo.ca/pgupta45/Sec007/Section007.html

Appendix D. Trigonometry

On the triangle.



$$a^2+b^2=c^2$$

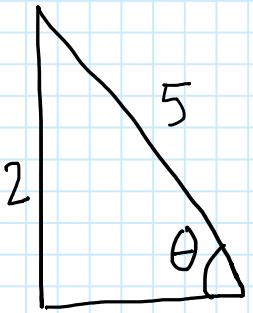
$\sin\theta = \frac{a}{c}$ $\quad \operatorname{cosec}\theta = \frac{c}{a}$

$\cos\theta = \frac{b}{c}$ $\quad \sec\theta = \frac{c}{b}$

$\tan\theta = \frac{a}{b}$ $\quad \cot\theta = \frac{b}{a}$

$0 < \theta < 90^\circ$

Ex: $\sin\theta = \frac{2}{5}$. $0<\theta<90^\circ$. Determine the other trig. quantities for θ.



$X = \sqrt{21}$

$2^2 + X^2 = 5^2$

$4 + X^2 = 25$

$X^2 = 21$

$x = \sqrt{21}$

$\cos\theta = \frac{\sqrt{21}}{5}$

$\tan\theta = \frac{2}{\sqrt{21}}$

H.W. Write the rest!

We use radians. θ: d degrees, r radians

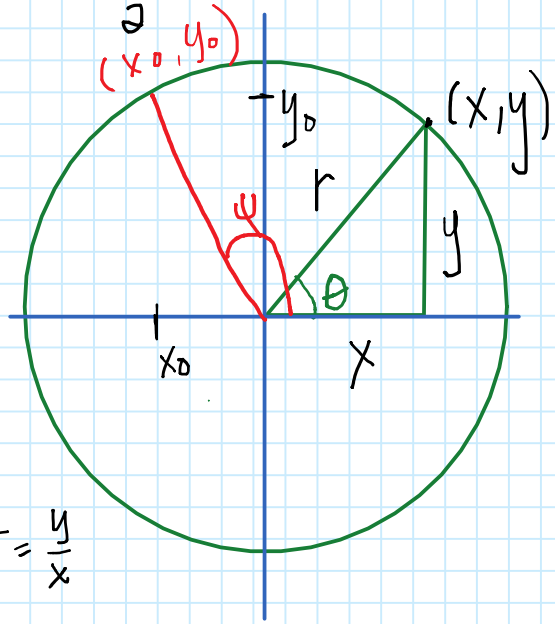
$d = \frac{r}{2\pi} \cdot 360$

$\cos(30)$ rad.

$\cos(30^\circ)$ deg

other trig. quantities for θ.

Trig. functions for angles $\geq \frac{\pi}{2}$



$$\sin\theta = \frac{y}{r}$$

$$\cos\theta = \frac{x}{r}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{x}$$

$$\psi > \frac{\pi}{2}$$

$$\cos\psi = \frac{x_0}{r} < 0 \qquad \sin\psi = \frac{y_0}{r} > 0$$

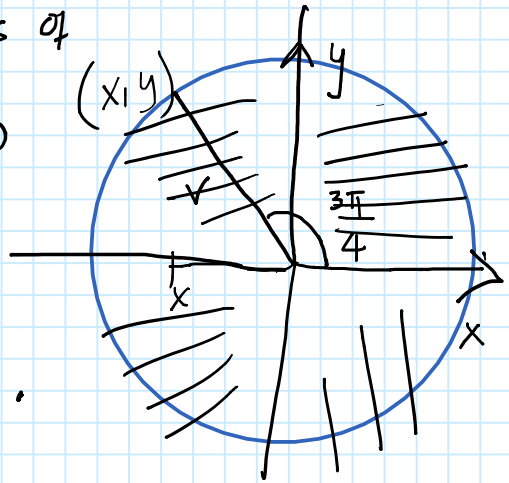
Q: Determine the sign of

Q Determine the signs of

(i) $\cos\left(\frac{3\pi}{4}\right) = \frac{x}{r} < 0$

$\frac{1}{2}\pi < \frac{3}{4}\pi < 1\pi$

$\cos\left(\frac{3\pi}{4}\right) = \frac{x}{r} < 0.$

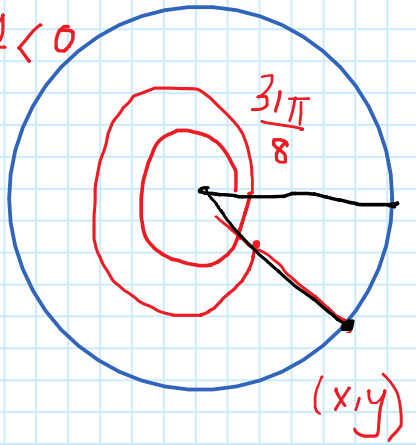


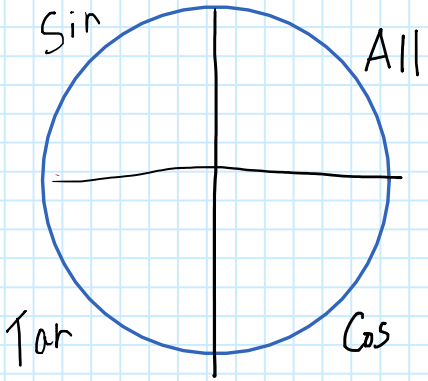
(ii) $\tan\left(\frac{31\pi}{8}\right) = \frac{\sin\left(\frac{31\pi}{8}\right)}{\cos\left(\frac{31\pi}{8}\right)} = \frac{y}{x} < 0$

$\frac{31}{8}\pi > 2\pi$

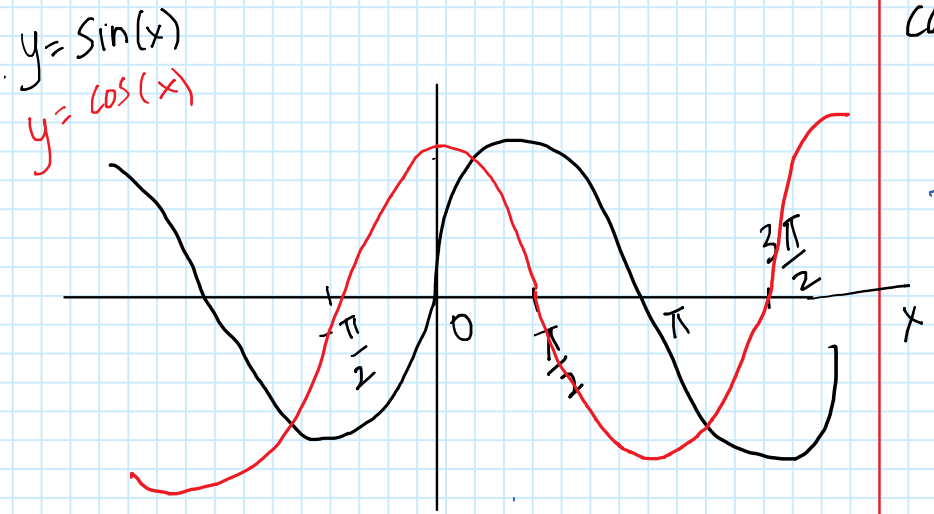
$\frac{31}{8}\pi - 2\pi = \frac{15\pi}{8}$

$= \frac{16\pi - \pi}{8} = 2\pi - \frac{\pi}{8}$





http://www.analyzemath.com/unitcircle/unit_circle_applet.html



$\sin(x)$ is periodic with period 2π

$\cos(x)$ " " " " "

$$\sin(x+2\pi) = \sin(x)$$

$$\cos(x+2\pi) = \cos(x)$$

$\sin(x)$ is an odd fn.

$$\sin(-x) = -\sin(x)$$

$\cos(x)$ is an even fn

$$\cos(-x) = \cos(x).$$

INTERVAL NOTATION

$1 \le x \le 2 \rightsquigarrow [1,2]$

$1 < x < 2 \rightsquigarrow (1,2)$

$1 \le x < 2 \rightsquigarrow [1,2)$

$1 \le x$

$x < 2$

$[1,\infty)$

WW

$[1, \text{Inf})$

INTERVAL NOTATION

$1 \le x \le 2 \rightsquigarrow [1,2]$

$1 < x < 2 \rightsquigarrow (1,2)$

$1 \le x < 2 \rightsquigarrow [1,2)$

$1 \le x \rightsquigarrow [1,\infty)$

$x < 2 \rightsquigarrow (-\infty, 2)$

WW $([1, \text{Inf}))$

WW $((-\text{Inf}, 2))$

domain of sin, cos $\} (-\infty, \infty) \rightsquigarrow$ all real numbers

Range of sin, Range of cos $\} [-1,1]$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$$

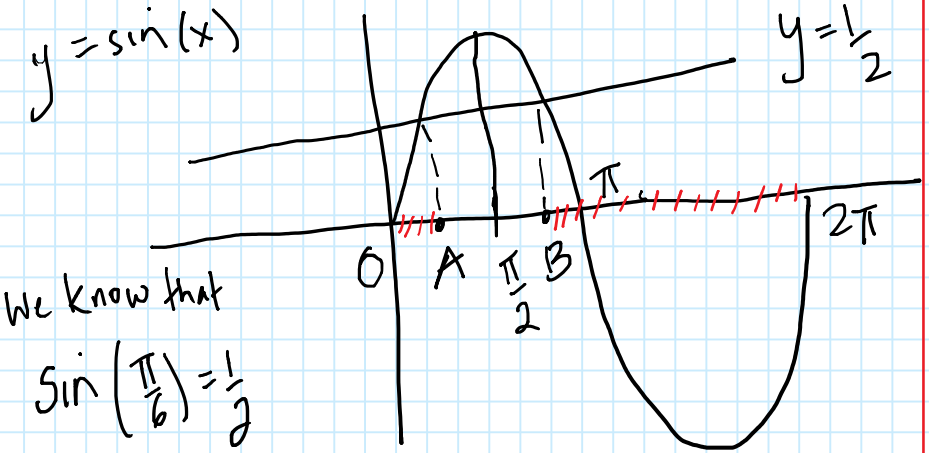
$$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$$

$$\sin(\pi + x) = -\sin(x)$$

$$\cos(\pi + x) = -\cos(x).$$

Ex. Solve for all values of x in $[0, 2\pi]$ such that

$$\sin(x) \leq \frac{1}{2}.$$



We know that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$A = \frac{\pi}{6}$

Use $\sin(\pi - x) = \sin(x)$

$\sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$B = \frac{5\pi}{6}$

$$\left[0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$$

$$0 \leq x \leq \frac{\pi}{6} \quad \& \quad \frac{5\pi}{6} \leq x \leq 2\pi.$$

Ex.
- periodicity of $\tan(x)$
- is $\tan(x)$ odd or even.
- domain & range $\tan(x)$.

} to think about

$f(x) = \sin(x) + 3$ Domain ?

Range ?

$\sin(x) + 3$ is $\sin(x)$ shifted up by 3 units.

domain: $(-\infty, \infty)$

Range: $[2, 4]$

Range sin $[-1, 1]$

Trigonometric identities

$\sin^2\theta + \cos^2\theta = 1$ ★

Divide ★ $\cos^2\theta$

$\tan^2\theta + 1 = \sec^2\theta$

Addition

$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

Subtraction

$\sin(x-y) = \sin(x + (-y))$

$= \sin(x)\cos(-y) + \cos(x)\sin(-y)$

$= \sin(x)\cos(y) + \cos(x)(-\sin(y))$

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$.

Ex $\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$

$\sin(2x) = \sin(x+x)$

$= \sin(x)\cos(x) + \cos(x)\sin(x)$

$= 2\sin(x)\cos(x)$

Ex. Prove:

$$\frac{\sin\phi}{1-\cos\phi} = \operatorname{cosec}\phi + \cot\phi$$

R.H.S. $\dfrac{1}{\sin\phi} + \dfrac{\cos\phi}{\sin\phi}$

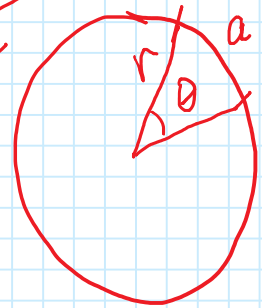
$$= \frac{1+\cos\phi}{\sin\phi} \times \frac{(1-\cos\phi)}{(1-\cos\phi)}$$

$$= \frac{1-\cos^2\phi}{\sin\phi\,(1-\cos\phi)} = \frac{\sin^2\phi}{\sin\phi\,(1-\cos\phi)}$$

$$= \frac{\sin\phi}{1-\cos\phi} = \text{L.H.S.}$$

$(a-b)(a+b)$
$= a^2 - b^2$

Read up



arc-length formula:

$a = r\theta$

↳ radians