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Math 640
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Homework 12

Problem 3

Proof. The proof will be by induction on n. We see that the coefficient on x^0 is 1, so we have $c(0) = 1$, which is an integer. Now, assume that $c(0)$ through $c(n-1)$ are integers. In order to find $c(n)$, we equate coefficients in $y_n' = f(x, y_n)$, where

$$y_n = \sum_{i=0}^{n} \frac{c(n)}{n!} x^n.$$

Since $y_n = y_{n-1} + \frac{c(n)}{n!} x^n$, this gives us the equation

$$n \frac{c(n)}{n!} = \text{coefficient of } x^{n-1} \text{ in } f\left(x, y_{n-1} + \frac{c(n)}{n!} x^n\right).$$

The left side equals

$$\frac{c(n)}{(n-1)!},$$

and by induction and the fact that the desired coefficient does not depend on terms of degree higher than x^{n-1} the right side is an integer of $(n-1)!$. Therefore, $c(n)$ is an integer, as required. □