Quiz 5

Name (please PRINT):

Out of fairness to your classmates, please stay on this page and <u>DO NOT BEGIN</u> until told otherwise.

Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. <u>**Absolutely no writing**</u> after time is called.
- Write your printed name on all sheets containing work.
- $\boxed{\text{Box}}$ your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Score: (curved, out of 20)

Question 1 Give an example, with proof, of each of the following categories of graph, or prove that no such graph exists:

1. A graph G that is both Hamiltonian and Eulerian
   Example answer: K_n for n odd, because all its vertices have even degree (Eulerian) and it contains an $(n-1)$-cycle (Hamiltonian).

2. A connected graph G that is neither Hamiltonian nor Eulerian
   Example answer: Any star, *i.e.*, , tree with $n-1$ vertices with degree 1 and single vertex of degree $n-1$, because it contains no cycles whatsoever (so cannot be Hamiltonian or Eulerian).

3. A non-trivial graph G such that both G and $\overline{G}$ are Eulerian
   Example answer: The cycle of length 5, because all its vertices have degree 2 (so it is Eulerian) and it is isomorphic to its complement (which is therefore also Eulerian).

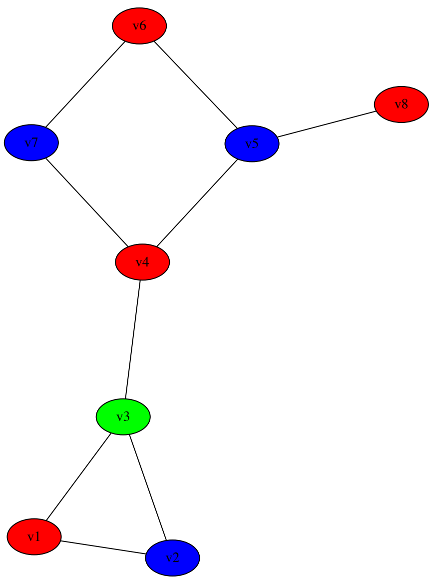
Question 2 Recall that we proved the inequality

$$\chi(G) \leq 1 + \Delta(G)$$

using the method of *greedy coloring*.

(a) Use the same method to color the following labelled graph G, then prove that this coloring uses $\chi(G)$ colors:

Answer You should obtain the below coloring, proving that $\chi(G) \leq 3$. On the other hand, since G contains a triangle, which is an odd cycle, G is not bipartite and so $\chi(G) \geq 3$. Thus $\chi(G) = 3$.



(b) Find $\chi'(G)$.

Answer Clearly $\chi'(G) \in \{\Delta(G), \Delta(G) + 1\} = \{3, 4\}$. On the other hand, we obtain the following 3-coloring through a greedy coloring in which the colors are ranked *red*, *blue*, and *green*; edges incident to vertices of highest degree are colored first; and of two edges incident to a certain vertex, the one incident to the second vertex with lower index will be colored first.