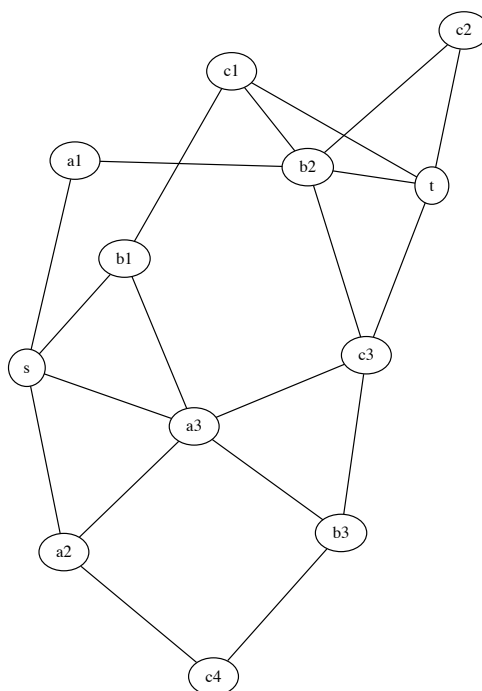


Quiz 4

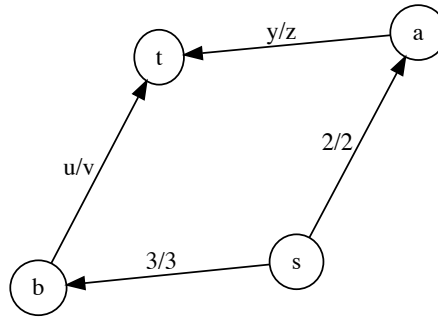
Question 1 Consider the following graph G :



Find, with support, the values $\kappa(G)$ and $\lambda(G)$.

Answer By the connectedness of the graph, $1 \leq \kappa(G) \leq \lambda(G)$. On the other hand, $\lambda(G) = 1$ if and only if G has a bridge; since every edge is on a cycle, $\lambda(G) \geq 2$. However, $\lambda(G) \leq \delta(G) = 2$, implying $\lambda(G) = 2$. Finally, the partition of the edge set of a graph into blocks is an equivalence relation; thus every two edges of G lie in a common cycle and G itself is a maximal 2-connected subgraph of itself. Thus $\kappa(G) = \lambda(G) = 2$.

Question 2 In the below digraph, arcs (*i.e.*, directed edges) are labelled *flow/capacity* with orientation in the direction of the arrow. Capacity on unlabelled arcs is zero, and the flow may be assumed to be maximal.



Find the following values (exact values are required, no variables):

- $|f|$
5: by definition, $|f| := f(s, V) = f(s, a) + f(s, b) = 2 + 3 = 5$.
- $f(t, V)$
-5: axiomatically, $f(y, x) = -f(x, y)$ and thus $f(t, V) = f(V, t) = |f|$.
- $f(b, t)$
3: axiomatically, $f(b, V) = 0$ and thus $f(b, t) = -f(b, s) = f(s, b)$.
- $f(t, a)$
2. Combine two axioms: $f(a, V) = 0$, so $f(a, t) = f(a, s) = -2$, so $f(t, a) = 2$.