Quiz 3

Name (please PRINT):

Out of fairness to your classmates, please stay on this page and <u>DO NOT BEGIN</u> until told otherwise.

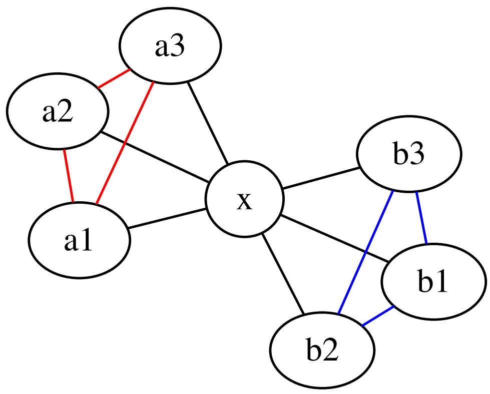
Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. <u>**Absolutely no writing**</u> after time is called.
- Write your printed name on all sheets containing work.
- $\boxed{\text{Box}}$ your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Score: (curved, out of 20)

Question 1 Draw examples of the following or explain why none exist:

- An acyclic graph of order 7 with 4 edges
  Any forest with 3 components will do.
- A 1-connected graph that is also 2-connected.
  Any 2-connected graph will do.
- A 2-edge-connected graph that is also 1-edge-connected.
  Any 2-edge-connected graph will do.
- A tree T with the property $\kappa(T) < \lambda(T)$.
  IMPOSSIBLE. A graph has $\lambda(T) \geq 2$ if and only if it has no bridge, so any tree has $\lambda(G) = 1$. But if $\kappa(G) = 0$ then G is disconnected by definition, contradiction the definition of a tree as a *connected* acyclic graph.
- A graph G of order 5 such that $\lambda(G) = 4$.
  The complete graph K_5, by definition.
- A graph H such that $\kappa(H) = 1$ yet $\lambda(H) = 3$.
  Consider the following graph:



Question 2 Let G be the graph K_4 whose vertices are labelled with $\{1, 2, 3, 4\}$. How many spanning trees does $G - \{e\}$ contain, where e is the edge $\{2, 3\}$?

Answer The Lagrangian of this graph is its degree matrix minus its adjacency matrix:

$$L = \begin{pmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{pmatrix}$$

Here are two submatrices of L, each containing two zeros:

$$L_{1,1} = \begin{pmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ -1 & -1 & 3 \end{pmatrix} \quad L_{1,4} = \begin{pmatrix} -1 & 2 & 0 \\ -1 & 0 & 2 \\ -1 & -1 & -1 \end{pmatrix}$$

These matrices yield determinants (*i.e.*, minors) of 8 and -8 respectively, corresponding to a cofactor (signed minor) of 8. Thus there are eight labelled trees in the graph.