Quiz 2

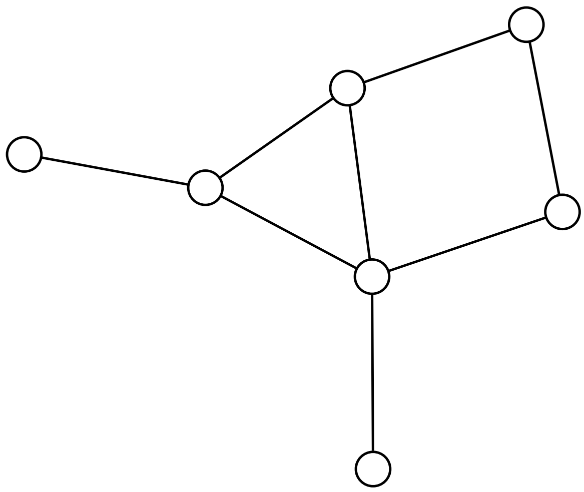
Name (please PRINT):

Out of fairness to your classmates, please stay on this page and <u>DO NOT BEGIN</u> until told otherwise.

Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. **<u>Absolutely no writing</u>** after time is called.
- Write your printed name on all sheets containing work.
- $\boxed{\text{Box}}$ your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Question 1 Consider the following graph G:



1. Is G bipartite? If so, provide a partition of its vertices into appropriate sets; if not, rigorously explain why not.
2. Define the *diameter* of a graph and find the diameter of G.
3. Find the quantities $\delta(G)$ and $\Delta(G)$.
4. Give the order and size of G.

You may use this sheet for work.

Directions Answer AT LEAST ONE of questions 2 and 3. If you attempt to answer both, clearly indicate which one should be graded first: if you earn at least 80% credit on this one then points for the other will count for extra credit in the Quizzes category.

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Question 2 Prove or disprove that each of the following relations is an equivalence relation:

- On the set $\mathbb{Z}$ of integers, $m \sim n$ if and only if m and n are either both odd or both even (*i.e.,* iff they have *the same parity*)
- On the set $V(G)$ of vertices in an arbitrary graph G, $u \sim v$ if and only if u and v are NOT adjacent.

You may use this sheet for work.

Question 3 Recall that the graph $G[S]$ is the induced subgraph of G on the subset S of $V(G)$. Prove that the vertices of every connected graph of order n can be labelled in an order

$$\{v_1, ..., v_n\}$$

such that the induced graph

$$G[v_1, ..., v_k]$$

is connected for every k with $1 \leq k \leq n$.

You may use this sheet or its reverse for work.