Quiz 1

Name (please PRINT):

Out of fairness to your classmates, please stay on this page and DO NOT BEGIN until told otherwise.

Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. **Absolutely no writing** after time is called.
- Write your printed name on all sheets containing work.
- Box your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Score: (curved, out of 20)

Question 1 Suppose G is a connected graph of order n. What are the greatest and least possible sizes of G? Fully justify both answers.

You may use this sheet for work.

Question 2 Prove that every graph G has a path of length at least $\delta(G)$.

HINT: Let $P = x_1x_2...x_k$ be the **longest path** in G. Combine an inequality between $\delta(G)$ and $|\Gamma(x_k)|$ with the fact that P *cannot be extended*.

You may use this sheet or its reverse for work.