Name (please PRINT):

Out of fairness to your classmates, please stay on this page and DO NOT BEGIN until told otherwise.

Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. **Absolutely no writing** after time is called.
- Write your printed name on all sheets containing work.
- $\boxed{\text{Box}}$ your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Final Exam, Part II
80 minutes

Score: (curved, out of 45)

Prob. No.	Grade?	Earned	Total
1			15
2			15
3			15
4			15
5			15
Part II Total			45

Question II [45 pts] Answer THREE of the following FIVE questions, making sure to clearly indicate your choices on the rubric.

1. State Menger's Theorem (edge form), then use the Max-Flow/Min-Cut Theorem to prove it.

2. State Menger's Theorem (vertex form), then use the Max-Flow/Min-Cut Theorem to prove it.

   DEFINITION 1 (CUBIC GRAPH):
3. A graph G with the property $\delta(G) = \Delta(G) = 3$.

   Prove that every cut-vertex of a cubic graph G is the endpoint of a bridge of G.

4. Is it possible to choose in a sequence, for each integer n from 101 to 200, a proper divisor of n in such a way that we never repeat a divisor previously chosen?

5. Let $m \neq 0$ denote the size and n the order of a certain graph G.

   Prove that
   $$\lambda(G) \leq \lceil 2m/n \rceil$$
   and determine whether or not the bound is sharp; if it is not, find the optimal bound, if it is, prove the bound's sharpness.