Name (please PRINT):

Out of fairness to your classmates, please stay on this page and DO NOT BEGIN until told otherwise.

Sloppy handwriting increases the chance of grading errors: please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between. Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Close bags and silence electronics – this quiz is closed-resource.
- If you are still working when time is called, you must stop immediately and bring your quiz to the front. **Absolutely no writing** after time is called.
- Write your printed name on all sheets containing work.
- Box your final answers.
- As much as possible, rubrics are designed so that your grade will not "cascade" from a mistake early in a problem: move on if you have trouble for too long in an early subproblem.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

Practice Final Exam, Part I
80 minutes

Score: (curved, out of 55)

Prob. No.	Grade?	Earned	Total
A	(obligatory)		14
B	(obligatory)		15
C1			13
C2			13
C3			13
C subtotal			26
Part I Total			55

Question I (A)

Categorize each of the following statements as TRUE or FALSE, where TRUE means "true without further restrictions."

1. _____ If $H \subset G$ is an induced subgraph of G then $\chi(H) \leq \chi(G)$.
2. _____ $\chi(G) = 2$ if and only if G is bipartite.
3. _____ The graph $C_4 + C_4$ is perfect.
4. _____ The edge chromatic number of a graph equals the chromatic number of its line graph.
5. _____ No tree is Eulerian.
6. _____ Every tree contains a Hamiltonian path.
7. _____ A graph is Eulerian if and only if it contains a cycle consisting of all its edges.

Question I (B)

In the below, assume $|G| \geq 2$.

Consider the statement P_3, defined as

> For any $x, y, z \in G$, there exists an $x - z$ path containing y.

Does P_3 provide a **necessary** or **sufficient** condition for $\kappa(G) > 1$? Or both, or neither? Justify rigorously.

You may use this sheet for work.

Question I (C)

Choose TWO of the following three questions to answer, clearly indicating your choice both below and on the score sheet. You may use the remaining blank sheets for work.

1. _____ Briefly describe the labelled tree on $[n]$ whose Prüfer code is

$$\underbrace{n, n, \ldots, n}_{n \text{ repeated } n-2 \text{ times}}$$

2. _____ Prove the multi-colour Ramsey theorem – *i.e.,* , prove that, for any k colours $c_1, c_2, ..., c_k$ there exists a number $r = R(n_1, n_2, ..., n_k)$ such that any colouring of the edges of K_r with k colours will contain a c_1-coloured n_1-clique, or a c_2-coloured n_2-clique, ..., or a c_k-coloured n_k-clique.

3. _____ Prove that

$$R(4, 3) \leq 9$$

without using a closed-form identity (*e.g.*, a binomial coefficient or power of 2).

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Please make sure your solutions are complete before time is called.

YOU CANNOT COME BACK TO THIS PART AFTER THE BREAK.