Quiz 6

Score: (curved, out of 20)

Consider the system

$$dx/dt = 2x + y + xy^3$$
$$dy/dt = x - 2y - xy$$

Determine its critical points, then write the the linear system near each critical point in matrix form. Finally, find the eigenvalues corresponding to each critical point and use them to make a conclusion about the behavior about the non-linear system near each point.

Answer A correct answer for any one critical point is sufficient for full credit. For example,

At (0,0), $dx/dt = dy/dt = 0$. The linear system there is given by the Jacobian, which is determined by

$$F_x = 2 + y^3 = 2$$
$$F_y = 1 + 3xy^2 = 1$$
$$G_x = 1 - y = 1$$
$$G_y = -2 - x = -2$$

The eigenvalues at $(0, 0)$ are the solutions of $(2-r)(-2-r)-1 = 0 = -5+r^2$, or $r = \pm\sqrt{5}$. Thus, near the origin the non-linear system behaves like an unstable linear system, with one line of solutions asymptotically approaching the origin and all others leaving it.

Points breakdown **16 points** for any reasonable attempt to answer the question (*i.e.,* anything other than a doodle), **20 pts** for a fully correct answer, 1 point over 16 for each correct prong.