Take-home quiz 3

Name (please PRINT):

Please write from TOP TO BOTTOM, moving columns of work from LEFT TO RIGHT with STRAIGHT MARGINS in between.

Ensure that no work is overlooked by clearly marking any point at which you make an exception to these guidelines.

- Write your printed name on all sheets containing work.
- Box your final answers.
- While you generally need not write in short essay form, you must demonstrate knowledge of course material, supplementing your mathematical notation with words if necessary. In particular, you must
  - *explicitly* cite any theorems you use from the course and
  - write conclusions using at least a few words.

This is an individual task.
Collaboration and the use of online forums are prohibited.

You may consult other works than your textbook for help but must cite each instance that you do so.

Score: (curved, out of 20)

Question 1 [10 pts]

At the end of a spring with constant $2\ N/m$ is a $1\ kg$ mass. The spring vibrates in a medium such that the damping constant is $\gamma = 3\, kg/s$, and the mass is attached to a device which exerts on it a force expressed by

$$F(t) = 130e^{-3t}\sin(3t)$$

(*i.e.,* $F(t)$ is the forcing term in the differential equation of this spring).

(a) Give the general solution $u(t)$ for the position of the mass attached to this spring.

(b) Is it possible to determine, from the general solution alone, the behavior of the amplitude of such a mass as $t \to \infty$? Explain why or why not.

You may use this page for work.

Question 2 [10 pts]

A 2 kg mass without damping is pulled down on a spring with constant 2 N/m. It is released and, when observation begins (at $t = 0$), is clocked moving upward at 1 m/s from a position 1 m below the equilibrium point.

Write an equation for the position of this mass in the form

$$u(t) = R\cos(\omega t - \delta)$$

then graph this function in a way that accurately portrays the amplitude, frequency, phase, and end behavior of the mass in time.

You may use this page and its reverse for work.