

244 Review, Day 4 (no hints)

Question IX

Find the Taylor series of e^x at $x = 0$ using only the following two facts:

- e^x is analytic at (*i.e.*, can be expressed by a Taylor series in an open neighbourhood of) $x = 0$;
- $y = e^x$ is the unique function equal to its own **first** derivative and such that $y(0) = 1$.

(*i.e.*, you should NOT directly use any value or property of an n^{th} derivative of e^x in your answer)

Question X

Derive the formulas for one-variable integrating factors for exact equations, first in x and then in y .

Question XI

Ascertain, with justification, which of the adjectives {autonomous, separable, exact} apply to the differential equation

$$x + y \frac{dy}{dx} = 0; \quad y(0) = -1$$

Use the methods of separable/exact equations, as applicable, to solve for an explicit solution $y = f(x)$ to the equation; if both apply, use one to solve and the other to check your answer. (See HINTS 1-3, next page.)

HINT 1. The question requires an EXPLICIT solution. Please read the directions carefully.

HINT 2.

What does it mean for a DE to be exact?

Are you sure?

Is that exactly the definition (pun intended)?

Or are you paraphrasing (*e.g.*, confusing a theorem *applicable* to exact differential equations with the *definition* of exact differential equations)?

(You are of course not required to appeal to the technical definition of *exact* and to the contrary may use any mathematically legal means of proving or disproving exactness – just make sure you are being precise and thorough.)

HINT 3. Many of the same pitfalls from Question 5 of Exam I are built in to the solution of this problem – review that problem if you haven't seen it in a while. In particular, do not confuse integration and derivation and be careful with roots.