244 Review, Day 1 (no hints)

Question I

Find a family of functions $\{y_m(t)\}_{m\in\mathbb{Z}}$ with the following properties:

- At each point t, multiplying $y_m(t)$ by m and adding 1 yields the second derivative of y_m at t
- For all integers $m, y_m(0) = 0$
- For all integers $m, y'_m(0) = 1$

Question II

Solve the differential equation

$$y' + 2\tan(t) \cdot y = 3\tan^2(t), \quad y(3\pi/4) = -1/2$$
 (1)

Question III

Customers A, B, and C have applied for five-year automobile loans. For each scenario, write a differential equation (with any necessary initial conditions), then give an answer to the nearest dollar (or, for part (c), to the nearest one-hundredth of a per cent).

Assume that payments are made, and interest accrued, continuously, and that only five-year loans are available; also ignore other factors like down payments.

- Customer A has been approved for a loan with 8% annual interest and can afford a maximum \$300 monthly auto loan payment. What is the price of the most expensive car she can buy?
- Customer B has been approved for a loan on a \$20000 car with 6% annual interest. How much will his monthly payment be?
- Customer C has not yet been approved for a loan, but wants to buy a \$15000 car and can only afford to pay \$300 per month towards auto loans. What is the maximum annual interest rate offer she can accept?