

* Users online: TAMatth

* Bry joined

Bry Good morning

TAMatth Hi! Feel free to ask questions; this is really just a Q&A session.

TAMatth Good morning!

Bry Okay I was waiting for anyone else

TAMatth Yeah, don't worry about that. Attendance isn't mandatory and I'm not going to lecture -- just answer questions.

Bry I just would like more of an explanation of Existence and Uniqueness Thm

TAMatth Can you be a bit more specific? Do you want an example? An explanation of the necessary hypotheses? etc.

Bry I guess an example would be the best

TAMatth Do you have Matlab installed?

TAMatth Or Apple's proprietary grapher?

Bry No but for next recitation I can have Matlab

TAMatth Okay. It would be best, just in case I give an example that requires visuals.

TAMatth <https://bluffton.edu/homepages/facstaff/nesterd/java/slopefields.html> << this will do for now

Bry Could you review 5-8 in section 1.5 if you have the textbook next to you ?4

TAMatth OK, do you want the example first?

Bry yes

TAMatth Ok

TAMatth Consider the DE

$$\frac{dy}{dt} = \sqrt{|y|}$$

TAMatth with initial condition $y(0) = 0$

TAMatth If we call $f(t, y) = \sqrt{|y|}$ (cf. page 64) ...

TAMatth Then f is a continuous function of t and y in a rectangle (in fact, ANY rectangle) containing $(0, 0)$...

TAMatth And so the Existence Thm applies and we have $\epsilon > 0$ such that there is a function y from $(-\epsilon, \epsilon) \rightarrow \mathbb{R}$...

TAMatth that satisfies the differential equation. We did *NOT* say that this solution was unique; there may be more than one ...

TAMatth And in fact there are multiple solutions with these initial conditions ...

TAMatth QUIZ: How can you tell?

Bry By looking at the slope field

Bry ?

TAMatth Yes, and by noticing that $y = 0$ is an equilibrium solution.

TAMatth If you graph this DE in Matlab or Grapher ...

TAMatth you can probably notice that this eqn has two solutions with ics $(0,0)$...

TAMatth One is the equilibrium solution ...

TAMatth And the other is $y = x * |x|$.

TAMatth (should technically be ts but you get the idea).

TAMatth QUIZ: Why is the solution not unique? I.e., what part of the Uniqueness Thm fails, and why?

Bry Because the DE intersects the t axis two times and the partial derv is not continuous

Bry Nvm

Bry it doesnt intersect twice

Bry so its because the partial derv is not continuous

TAMatth That's correct; the PD has a denominator which vanishes at $y = 0$ and thus isn't continuous there. ...

TAMatth Okay, now for those questions.

Bry I understand

TAMatth But first: any questions over what we've gone over?

Bry no none so far

TAMatth I agree that the wording of this question is confusing ...

TAMatth All of the solutions will exist & be unique because $f(t,y)$ is a polynomial, thus continuous w/ continuous PD ...

TAMatth The book wants you to think end behaviour -- what does each solution look like as $x \rightarrow \infty$ or $x \rightarrow -\infty$?

TAMatth ... How could you make conclusions about the functions at $\pm\infty$ based on the theorems?

Bry I do not know

Bry Does it have to do with the equilibrium solutions

TAMatth Yes

TAMatth Also think about the sign of the derivative.

TAMatth Let's do #5 together ...

Bry okay

TAMatth Okay, so first of all our equilibrium solutions are

$$y = 0, 1, 3$$

TAMatth Also, dy/dt is trivially continuous everywhere w/ trivially continuous PD in y .

TAMatth (b/c it's a polynomial)

TAMatth Thus, for any set of initial conditions, the sol'n will both exist and be unique by the Existence and Uniqueness theorems.

TAMatth In particular -- and this is the key to the problem -- solutions cannot cross the asymptotes

$$y = 0, 1, 3$$

If they could then we would not have uniqueness ...

TAMatth at the crossing points. Okay, so for #5, since $y = 4$ at $t = 0$ our derivative is going to be positive ...

TAMatth In fact, it'll be positive at all points of our function since, as we just said, the function cannot cross the $y = 4$ horiz. asymptote.

Bry Isn't the asymptote at $y=3$

TAMatth Yes; sorry, that was a typo.

TAMatth We have, at $(0, 4)$, that $dy/dt = 4 * 3 * 1 = 12$...

Bry Np...If the the initial condition is $y(0)=-2$ or and negative the function will only be negative for all points because the asymptote at 0

TAMatth And so the function is increasing there. Since the derivative is not only positive but also increasing at points with positive values of t , we have that this function will increase without bound

TAMatth (To answer your question) Yes

TAMatth IF the hypotheses apply as they do in this case.

Bry OK, I see time is up.I must get ready for lecture... Thank you very much

TAMatth Ok. Is it OK if I post this conversation to Sakai, incl. your name?

Bry Yeah it is okay

TAMatth Great. See you soon!

* Bry left