What is a *linear* differential equation?

(Disclaimer: There is a much harder, more precise, and more technical definition out there that most grad students don't have the "tools" to fully understand, hence this definition is the correct one as of *right now, in this class.*)

A differential equation is **linear** when it can be expressed as a sum of the following form:

$$\sum_{k\in\mathbb{N}\cup\{0\}} a_k(t)\frac{d^k y}{dt^k} = f(t)$$

The function y CANNOT be used to calculate the functions a_k, as expressed by the fact that only t appears within the parentheses as an argument.

Examples of linear DEs:

$$y' + a(t)y = b(t) \tag{1}$$

$$t^2 y^{(100)} + ty'' + \sin(t)y = 0 \tag{2}$$

Non-examples:

$$(y')^2 + y = t \tag{3}$$

$$y' = \frac{t}{y^2} \tag{4}$$

In (1), the function y is a "zeroth derivative" of itself; in (3) the fact that the first derivative of y is squared disqualifies it from being linear. Note that (4) is separable but not linear.