Question 1 (7 pts) Solve the differential equation

$$\frac{dy}{dt} + y = 3e^{-t}$$

with initial condition $y(0) = 1$.

$p(s) = 1$

$\int p(s)\,ds = t$

$\mu(t) = e^{\int p(s)ds} = e^{t}$

$y(t) = \frac{1}{\mu(t)} \int \mu(s) q(s)\,ds$

$= e^{-t} \int e^{s} \cdot 3e^{-s}\,ds$

$= e^{-t}(3t + C)$

$1 = y(0) = 1 \cdot (3 \cdot 0 + C)$

$= C$

Hence

$y(t) = e^{-t}(3t+1)$

Question 2 (4 pts) Give the order of the following differential equations and indicate if they are linear or non-linear. You do NOT need to justify.

(1) $y'' = y + t^2$

2nd linear

(2) $y' = ty^2$

1st non-linear

Question 3 (9 pts) The equation

$$\frac{dy}{dt} = y^2 t$$

is separable.

(a) Write the solution that passes through $y(0) = 1$ as a function of the form $y = f(t)$.

If $y=0$ then $\frac{dy}{dt}=0$ hence there is a constant soln $y=0$. So we can neglect it and hence divide safely by y.

$$\frac{dy}{y^2} = t\,dt$$

$$\int dy \cdot y^{-2} = \int t\,dt$$

$$-y^{-1} = \frac{t^2}{2} + C$$

IC $-1 = \frac{0^2}{2} + C = C$

$$\frac{1}{-y} = \frac{t^2-2}{2}$$

$$2 = y(2-t^2)$$

$$\boxed{\frac{2}{2-t^2} = y(t)}$$

(b) Find the maximum interval of validity of this solution.

The only concern is division by zero in the exp'n of $y(t)$. So $2-t^2 \neq 0$

$t \neq \pm\sqrt{2}$.

Hence $(-\sqrt{2}, \sqrt{2})$