# Chapter 1

# Homework Solutions (Even)

## Section 1.1

### 1.1.2

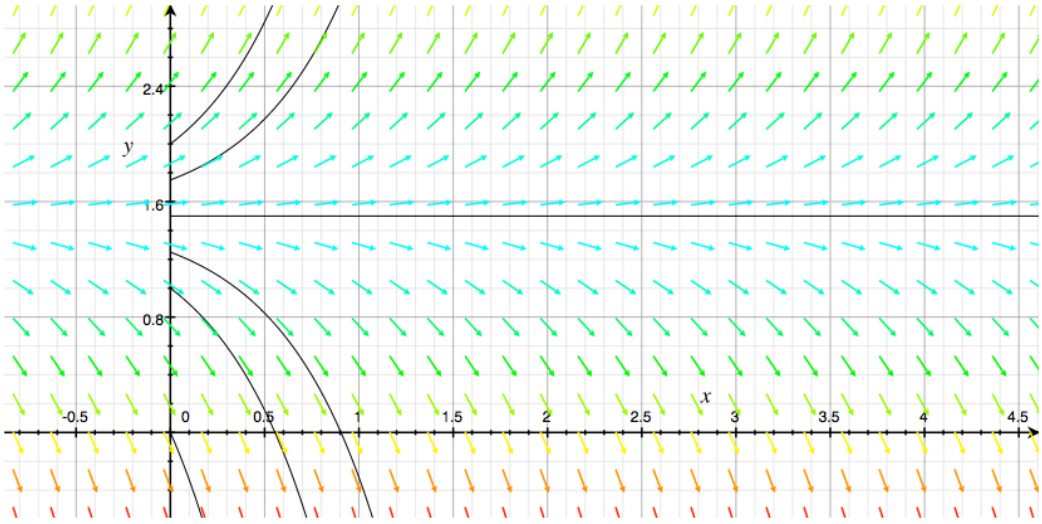


Figure 1.1: Direction field for Chapter 1.1, Problem 2.

When $y(0) \neq 3/2$, $y(t)$ diverges as $t \to \infty$; the initial condition $y(0) = 3/2$ corresponds to the constant solution $y = 3/2$.

# Section 1.2

## 1.2.2a

The formula gives $y = 5 + Ce^t$; a few solutions are plotted below:

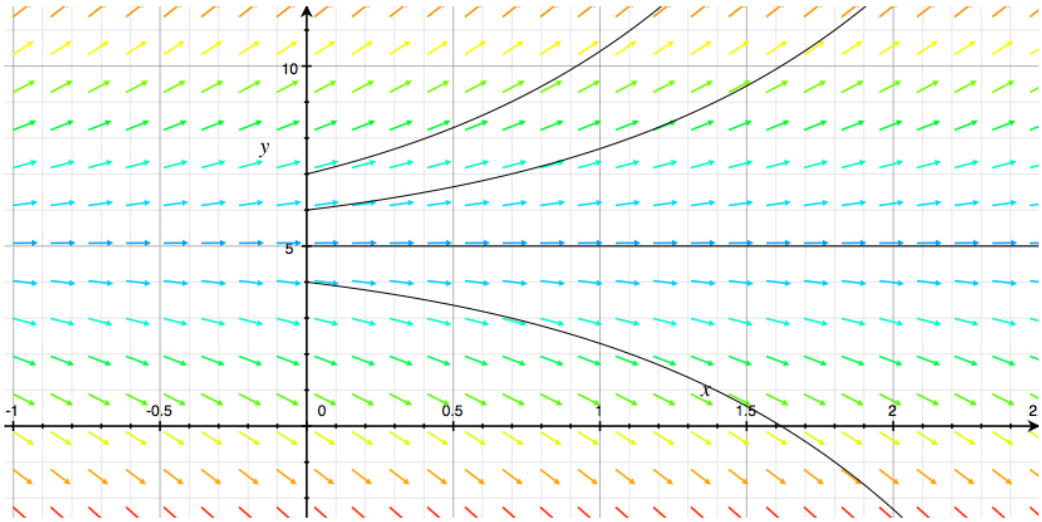


Figure 1.2: Direction field for Chapter 1.1, Problem 2.

All but the constant solution diverge.

## 1.2.12

$$Q(t) = Q(0)e^{-rt}$$

is the solution to the DE. If $t$ is given in weeks, then

$$Q(1) = (100 \text{ mg})e^{-r \cdot 1} \tag{1.1}$$
$$82.04 \text{ mg} = (100 \text{ mg})e^{-r} \tag{1.2}$$
$$-r = \ln(.8204) \tag{1.3}$$
$$r = -\ln(.8204) = 0.1980 \tag{1.4}$$

The half-life $t_{1/2}$ of thorium-234 is given by

$$\frac{Q(0)}{2} = Q(0)e^{-rt_{1/2}} \tag{1.5}$$

$$\ln 2 = rt_{1/2} \tag{1.6}$$

Hence

$$t_{1/2} = 3.5014 \text{ weeks} \tag{1.7}$$

# Section 1.3

## 1.3.4

First-order non-linear

## 1.3.6

Third-order linear

## 1.3.8

Yes

## 1.3.18

$$y''' - 3y'' + 2y' = 0 \tag{1.8}$$
$$r^3e^{rt} - 3r^2e^{rt} + 2re^{rt} = 0 \tag{1.9}$$
$$(r^3 - 3r^2 + 2r)e^{rt} = 0 \tag{1.10}$$
$$e^{rt} \neq 0, \text{ hence} \tag{1.11}$$
$$r^3 - 3r^2 + 2r = 0 \tag{1.12}$$
$$r(r-2)(r-1) = 0 \tag{1.13}$$
$$r = 0, 1, 2 \tag{1.14}$$